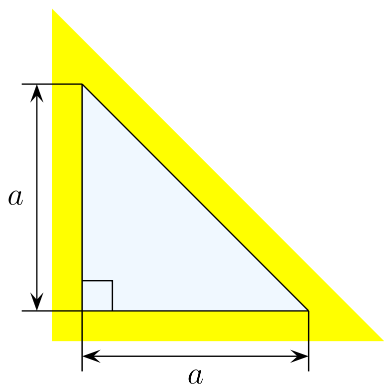
1. First a reading assignment: my notes on waveguides, the section about wave attenuation.

   (a) Pages 28-37 were explained in class on 10/29, but I went rather fast over many formulae, so it would help you to read them again but more carefully.

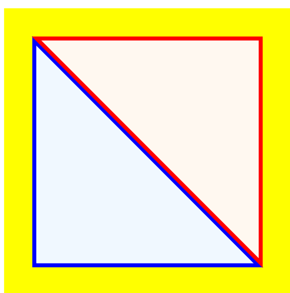
   (b) Pages 37-41, where I calculate the numerical coefficients F and G in eq. (175) for all the modes of a rectangular waveguide. This section was not covered in class, so read it carefully, as it would help you with problem 3 of this set.

2. Next, consider a waveguide with a triangular cross-section; specifically, the cross-section is a right isosceles triangle



Let's start by finding the eigenstates of the $-\nabla^2_{2\mathrm{d}}$ operator subject to Dirichlet or Neumann boundary conditions on all 3 sides of the triangle.

   (a) Let's mirror-reflect the triangle off its long side, so the original triangle plus its image form a square,



(1)

Suppose $\psi(x,y)$ obeys the eigenstate equation $(\nabla^2+\Gamma^2)\psi=0$ and the Neumann or Dirichlet boundary conditions on all 3 sides of the triangle. Let's continue this ψ to the whole square by mirror reflection off the diagonal side,

$$\psi(\mathbf{x}_{\mathrm{mirror}}) \;=\; \pm\psi(x_{\mathrm{orig}}) \tag{2}$$

where the sign is $+$ for the Neumann boundary conditions and $-$ for Dirichlet.

Show that the extended ψ obeys the eigenstate equation and the Neumann/Dirichlet boundary conditions for the whole square. In particular, show that $\psi(x, y)$ and its first derivatives are continuous across the diagonal line separating the original triangle from its image.

(b) Using the results of part (a), find all the eigenstates and the eigenvalues for the original triangle. Hint: look for linear combinations of the (m, n) and (n, m) eigenwaves of the square which happen to obey the appropriate boundary condition on the diagonal.

(c) Using the results of part (b), describe all the TM and TE waves of the triangular waveguide and calculate their cutoff frequencies. For simplicity, assume that the waveguide is filled with air or vacuum and let $\epsilon = \mu = 1$.

3. Continuing the previous problem of the triangular waveguide, suppose the waveguide's walls have small but non-zero resistivity $\rho = 1/\sigma$. This causes attenuation of the waves propagating down the waveguide, and your task is to calculate this attenuation rate for 2 specific waves:

   (a) The TE wave with the lowest cutoff frequency (among the TE waves).

   (b) The TM wave with the lowest cutoff frequency (among the TM waves).

   For simplicity, assume no dielectric inside the waveguide, thus $\epsilon = \mu = 1$.

4. Finally, consider a microwave resonator cavity in the shape for a cylinder of radius R and length d. For $d > 2.03R$, the lowest-frequency mode of this cavity is $\mathrm{TE}_{1,1,1}$. Calculate the quality factor Q of this mode.

   For simplicity, assume vacuum inside the cavity and neglect any possible opening in its surface. Also, assume all sides of the surface have the same surface resistivity R_s.

   Use Mathematica (or equivalent software) to calculate the integrals of Bessel functions and their derivatives.