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refraction index $n = \sqrt{\epsilon\mu}$

depends on ω

For ex, water ~~has~~ @ $\omega < 10^{9}\,s^{-1}$

has $\epsilon \approx 80 \Rightarrow n \approx 9$

but @ optical frequencies $\omega \sim 10^{15}\,s^{-1}$

water has $n \approx 1.33$

1) $n(\omega)$ depends on ω

2) $n(\omega)$ could be complex.

Im n causes Attenuation.

Plane wave $\vec{E}(\vec{x},t) = \vec{E}_0 e^{ikz - \omega t}$

$k = \frac{n(\omega)\,\omega}{c}$: complex for complex n

$k = k_r + i\varkappa$.

$k_r = \frac{\text{Re}\,n}{c}\omega$

$\varkappa = \frac{\text{Im}\,n}{c}\omega$

$\vec{E} = \vec{E}_0\, e^{ik_r z - \varkappa z - i\omega t}$.

Power $\vec{S} = \vec{E} \times \vec{B} \propto |E|^2$

$\boxed{S = S_0\, e^{-2\varkappa z}}$ Attenuation.

15(2) ω-dependence of $n(\omega)$, or even of $\mathrm{Re}\, n(\omega)$

$\to$ dispersion of wave packets

$\to$ slows down signal rate.

Medium, $\vec{D} = \varepsilon(\omega)\varepsilon_0\vec{E} = \varepsilon_0\vec{E} + \vec{P}$

$$\vec{E} = \vec{E}_0 e^{-i\omega t}$$

Suppose $\vec{P}(t)$ lags behind $\vec{E}(t)$

$$\vec{P}(t) = \vec{P}_0 e^{-i\omega(t-\delta t)}$$

$$= \vec{P}_0 e^{+i\omega\delta t} e^{-i\omega t}$$

$\vec{P} = \epsilon_0(\epsilon-1)\vec{E} \quad \to \epsilon-1 = |\epsilon-1| e^{i\delta t\omega}$

$\to$ complex $\epsilon-1$, $\Rightarrow$ complex $\varepsilon(\omega)$

$\boxed{\mathrm{Im}\,\varepsilon(\omega) > 0}$

Wave Power & work

$$\delta W = \iiint d^3x\, \vec{E}\cdot\delta\vec{D}$$

$$\text{Power} = \iiint d^3x\, \vec{E}\cdot\frac{\partial\vec{D}}{\partial t}.$$

$$\langle\text{Power}\rangle = \iiint d^3x\, \frac{1}{2}\mathrm{Re}\left[\vec{E}_0^{\,*}\cdot \underline{\underline{-i\omega\vec{D}_0}}\right].$$

15/3)

$$\vec{D}_0 = \varepsilon_0 \varepsilon \vec{E}_0$$

$$Re\left(E_0^* \varepsilon \varepsilon_0 (-i\omega E_0)\right) = |E|^2 \epsilon_0 \underbrace{Re\left(-i\omega \varepsilon(\omega)\right)}_{\omega \times Im\, \varepsilon(\omega)}$$

$$\langle \text{Power loss} \rangle = \frac{1}{2} \epsilon_0 \omega\, Im(\varepsilon(\omega)) \iiint d^3x\, |E_0|^2$$

$$\text{Energy density} = \frac{\epsilon_0}{2} Re(\varepsilon)\, |E_0|^2$$

$$\frac{\text{Power loss density}}{\text{Energy density}} = \frac{\omega\, Im\, \varepsilon}{Re\, \varepsilon}$$

$\hookrightarrow \frac{1}{4\pi} v \times$ attenuation rate $(2\varkappa)$

$$2\varkappa = \left(\frac{\omega}{v} = K\right) \times \frac{Im\, \varepsilon}{Re\, \varepsilon}$$

$$\frac{\varkappa}{K} = \frac{1}{2} \frac{Im\, \varepsilon}{Re\, \varepsilon} \approx \frac{Im\, n}{Re\, n}$$

for $n = \sqrt{\varepsilon}$

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Another source of EM power loss

→ conductivity

$$\vec{J}_{cond} = \sigma \vec{E}$$

→ Power loss density $= \vec{J}\cdot\vec{E} = \sigma \vec{E}^2$

$$\langle \text{Power loss density} \rangle = \tfrac{1}{2} \mathrm{Re}(\sigma) |\vec{E}_0|^2$$

net Loss density $= \frac{1}{2}|E_0|^2 \times (\mathrm{Re}(\sigma) + \omega \varepsilon_0 \mathrm{Im}(\varepsilon))$

Maxwell Ampere $\vec{\nabla}\times\vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$

net current $\vec{J}_{net} = \vec{J}_{cond} + \left(\vec{J}_{disp} = \frac{\partial \vec{D}}{\partial t}\right)$

$$= \sigma \vec{E} - i\omega\varepsilon_0 \varepsilon \vec{E}$$

$$= \sigma_c(\omega)\vec{E}$$

complex effective conductivity σ_c

$$\boxed{\sigma_c = \sigma(\omega) - i\omega\varepsilon_0\varepsilon(\omega)}$$

$$\boxed{\varepsilon_{eff}(\omega) = \frac{\sigma_c(\omega)}{i\omega\varepsilon_0} = \varepsilon(\omega) + \frac{i\sigma(\omega)}{\omega\varepsilon_0}}$$

→ power loss density $= \frac{1}{2}|\vec{E}_0|^2 \times \mathrm{Re}(\sigma_c)$

$$= \tfrac{1}{2}|E_0|^2 \times \omega\varepsilon_0 \mathrm{Im}(\varepsilon_{eff}(\omega))$$

15/5 Maxwell Eqs

$$\cancel{\nabla \times \vec{B} = -\frac{\partial E}{\partial t} = +i\omega \vec{E}}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = +i\omega \vec{B} = i\omega \mu_0 \vec{H}$$

$$\vec{\nabla} \times \vec{H} = 0 - i\omega \varepsilon_0 \varepsilon_{eff}(\omega) \times \vec{E}$$

$$\rightarrow \nabla \times \nabla \times H = \omega^2 \mu_0 \varepsilon_0 \epsilon_{eff}(\omega) \cdot \vec{H}$$

$$\begin{array}{c} \| \\ -\nabla^2 H \end{array} \rightarrow +k^2 \vec{H} \text{ for a plane wave}$$

$$k^2 = \frac{\omega^2}{c^2} \times \epsilon_{eff}(\omega)$$

$$\cancel{n^2 = c} \quad \boxed{n^2(\omega) = \epsilon_{eff}(\omega)}$$

including its imaginary part.

A conductor @ a low frequency

$$\rightarrow \epsilon_{eff} = \epsilon + \frac{i\sigma}{\varepsilon_0 \omega} \simeq \frac{i\sigma}{\varepsilon_0 \omega}$$

$$n \; \cancel{(\omega)} = \frac{1+i}{\sqrt{2}} \times \sqrt{\frac{\sigma}{\varepsilon_0 \omega}}$$

$$k = \frac{n\omega}{c} = (1+i)\sqrt{\frac{\sigma \omega \mu_0}{2}} = \frac{1+i}{\delta}$$

$\delta \equiv$ skin depth

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Another source of Power loss → attenuation

In a magnetic material, if magnetization lags behind the ext. mag. field.

→ $\vec{B} = \mu \mu_0 H$ with with complex $\mu(\omega)$

$\mathrm{Im}(\mu) > 0$.

→ $n^2(\omega) = \epsilon_{eff}(\omega) \times \mu(\omega)$

with ϵ_{eff} & μ containing imaginary parts.

Source of Power loss : magnetic work

$$\delta W = \iiint \vec{H} \cdot \delta \vec{B} \, d^3\bar{x}$$

$$\text{Power} = \iiint \vec{H} \cdot \frac{\partial \vec{B}}{\partial t} \, d^3\bar{x}$$

time-averaged $H \cdot \frac{\partial B}{\partial t} \to \bar{H} \times -i\omega \bar{B}$

$$\to \frac{1}{2} \mathrm{Re} (H^* \times -i\omega B^*)$$

$$= \frac{\omega}{2} \mathrm{Im} (H^* B = \mu \mu_0 |H|^2)$$

→ $\text{Power loss} = \frac{\mu_0}{2} \omega \, \mathrm{Im}(\mu) \times \iiint |H_0|^2 \, d^3\bar{x}.$