1. [Based on *Peskin and Schroeder* problem 10.2(b).] Continuing problem 2 from the previous homework#15, consider the Yukawa theory of a Dirac field $\Psi(x)$ and a real pseudoscalar field $\Phi(x)$, with the *physical Lagrangian*

$$\mathcal{L}_{\text{ph}} = \tfrac{1}{2}(\partial_\mu\Phi)^2 - \tfrac{1}{2}m^2\Phi^2 + \overline{\Psi}(i\partial\!\!\!/ - M)\Psi - ig\Phi\times\overline{\Psi}\gamma^5\Psi - \tfrac{1}{24}\lambda\Phi^4. \qquad (1)$$

In the previous homework, you should have argued that all the UV divergences of the theory can be canceled by just 6 counterterms with order-by-order adjustable coefficients δ_Z^ϕ, δ_m^ϕ, δ_Z^ψ, δ_m^ψ, δ_g, and δ_λ. In the present problem, your task is to calculate the divergent parts of all these counterterm coefficients to the one-loop order. For simplicity, do not worry about the finite parts of these counterterm coefficients.

Hint: the infinite part of the four-scalar amplitude $V(k_1,\ldots,k_4)$ does not depend on the scalar's momenta, so you may calculate it for any particular $k_1,\ldots,k_4$ you like, on-shell of off-shell. I suggest you take $k_1 = k_2 = k_3 = k_4 = 0$, so in any one-loop diagram all the propagators in the loop have the same momentum q — which makes evaluating such a diagram much simpler.

Likewise, the infinite part of the one-scalar-two-fermions amplitude $\Gamma^5(p',p)$ does not depend on the momenta p, p', or $k = p' - p$, so you may calculate it for any p and p' you like, on-shell or off shell. Again, letting $p = p' = 0$ makes for a much simpler calculation of the one-loop diagram(s).

Alas, this trick does not work for the two-scalar or the two-fermion amplitudes $\Sigma^\phi(k^2)$ or $\Sigma^\psi(p\!\!\!/)$: The divergent parts of these amplitudes do depend on the momentum, and you do need to know the momentum-dependence of these divergent parts to calculate the δ_Z^ϕ and δ_Z^ψ counterterms.

PS: Note that in the $\lambda_{\text{ph}} \to 0$ (but $g_{\text{ph}} \neq 0$) limit, the δ_λ counterterm does not vanish, so the bare Lagrangian has a non-zero 4-pseudoscalar coupling $\lambda_{\text{bare}} \neq 0$. On the other hand, in the $g_{\text{ph}} \to 0$ (but $\lambda_{\text{ph}} \neq 0$) limit, the δ_g counterterm — and hence the bare Yukawa coupling g_{bare} — do vanish along with the g_{ph}. This is an example of a general rule: *barring fine tuning of the coupling parameters, a renormalizable quantum field theory has all the renormalizable couplings consistent with the theorys symmetries*. Hence, *when some physical coupling happens to vanish, the corresponding bare coupling would also vanish only if in is absence the theory would have some extra symmetry.* For example, for $g = 0$ the Yukawa theory gets an extra symmetry $\Phi \to -\Phi$ (without space reflection), $\Psi \to \Psi$, so for $g_{\text{ph}} \to 0$ we also have $\delta_g \to 0$ and hence $g_{\text{bare}} \to 0$. On the other hand, there are no extra symmetries for $\lambda = 0$ (but $g \neq 0$), so taking $\lambda_{\text{ph}} \to 0$ would be a fine-tuning while δ_λ and hence λ_{bare} would not vanish along with the physical coupling.

2. And now a reading assignment: Study the two-loop example of a nested divergence in §10.5 of the *Peskin and Schroeder* textbook. This is a very hard calculation, so please read it very carefully and pay attention to details. In particular, pay attention to how the integral over the Feynman parameters gives rise to the second-order pole $1/\epsilon^2$ and to the momentum-dependent coefficient of the simple pole $1/\epsilon$.