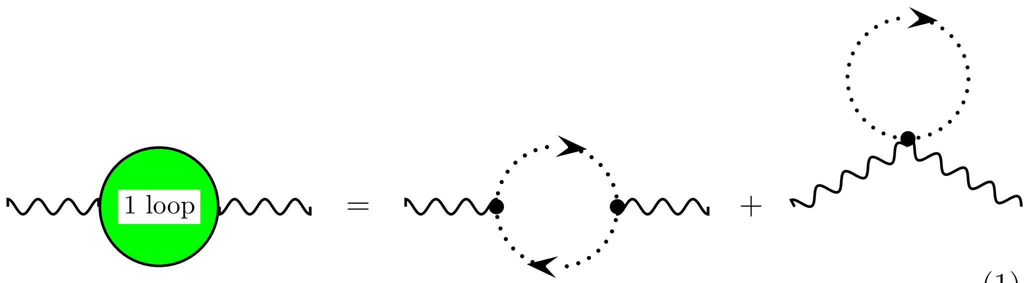
1. In class I have explained the electric charge renormalization in the basic QED. Now consider the *scalar QED* — the theory of the EM field A^μ interacting with a charged scalar field Φ. Specifically, let's work out the electric charge renormalization in this theory to the one-loop order. At this order, there are two Feynman diagrams contributing to the 1PI two-photon amplitude, namely

$i\Sigma^{\mu\nu}_{1\,\text{loop}}$ =



(1)

(a) Evaluate the two diagrams using dimensional regularization and verify that the net amplitude has form

$$\Sigma^{\mu\nu}_{1\,\text{loop}}(k) \;=\; \left(k^2 g^{\mu\nu} \,-\, k^\mu k^\nu\right) \times \Pi_{1\,\text{loop}}(k^2) \tag{2}$$

Note: the individual diagrams' amplitudes do not have this form. You need to add them up before the 'bad' terms cancel out.

(b) Calculate the $\Pi^{1\,\text{loop}}(k^2)$ due to two diagrams (1), add the δ_3 counter-term's contribution, then determine the $\delta_3^{\text{order}\,\alpha^1}$ coefficient — including its finite part, — and write down the *combined* $\Pi^{\text{net}}_{\text{order}\,\alpha^1}$ as a function of k^2.

(c) Consider the effective coupling $\alpha_{\text{eff}}(k^2)$ of the scalar QED at high off-shell momenta, $k^2 \gg m^2$. Show that at the one-loop level,

$$\frac{1}{\alpha_{\text{eff}}(k^2)} \;=\; \frac{1}{\alpha(0)} \;-\; \frac{1}{12\pi}\left(\log\frac{-k^2}{m^2} \,-\, \frac{8}{3}\right) \;+\; O(\alpha). \tag{3}$$

2. And now a big reading assignment: My notes on the diagrammatic proof of Ward–Takahashi identities. I shall go over these notes in class on Wednesday 2/12, but I shall skip over many technical details. So your task is to go *carefully* through the algebra, and make sure you understand what's going on.