1. Let's start with an exercise on 't Hooft's anomaly matching conditions. Consider an $SU(3)$ gauge theory with chiral fermions. In terms of LH Weyl spinors, they comprise six $\psi^{ij} = +\psi^{ji}$ in a symmetric tensor multiplet **6**, and 21 χ_i^f in seven 'flavors' of antifundamental $\bar{\mathbf{3}}$ multiplet.

   (a) Check that the $SU(3)$ gauge symmetry itself is anomaly free.

   The global continuous symmetry of this theory is $SU(7) \times U(1)$, where the $U(1)$ charges of the fermions are -7 for the ψ^{ij} and $+5$ for the χ_i^f, while the $SU(7)$ quantum numbers should be obvious.

   (b) Check that this global symmetry is free from the $SU(3)$ anomaly.

   The $SU(3)$ gauge symmetry has negative beta-function, so at some IR energy scale it becomes strongly interacting and confines. Suppose the confinement does not cause spontaneous breakdown of the chiral global symmetry $SU(7) \times U(1)$. Instead, the the spectrum of $SU(3)$-singlet composite hadrons includes a massless chiral baryon made from one ψ quark and two χ quarks. In terms of their gauge, flavor, and spin quantum numbers

$$B_\alpha^{ff'} \;=\; \psi^{ij,\beta}\bigl(\chi_{i,\alpha}^f \chi_{j,\beta}^{f'} \,+\, \chi_{i,\beta}^f \chi_{j,\alpha}\bigr). \tag{1}$$

   By construction, these baryons are in the antisymmetric **21** tensor multiplet of the $SU(7)$ and have $U(1)$ charge $+3$.

   (c) Check that these massless baryons obey the 't Hooft's anomaly matching conditions,

$$\mathrm{tr}_{\mathrm{baryons}}\bigl(t^a\{t^b,t^c\}\bigr) \;=\; \mathrm{tr}_{\mathrm{quarks}}\bigl(t^a\{t^b,t^c\}\bigr) \quad \forall a,b,c \in SU(7)\times U(1). \tag{2}$$

2. The rest of this homework is a reading assignment, 't Hooft's 1999 lecture notes, chapter 4. Read about the instantons and their effects from the Hamiltonian point of view.