E7/1

Green Schwarz in 4D.

Gauge group: several factors

$G = G_1 \times G_2 \times G_3 \times \cdots$

including abelian G_1.

$$\hat{Q}_6 = \mathrm{tr}_L(F \wedge F \wedge F) - \mathrm{tr}_R(F \wedge F \wedge F).$$

Suppose $\hat{Q}_6$ factorizes to

$$\hat{Q}_6 = F_1 \wedge \sum_a F^a \wedge F^a$$

or $F_1 \wedge Q_4$

$$Q_4 = \sum c_a F_a \wedge F_a$$

c_a: may be different for diff G_a

or $Q_4 = \sum_{\text{abelian}} c_{ab} F^a \wedge F^b$

$+ \sum_{\text{non-abelian}} c_a \, \mathrm{tr}(F \wedge F)_{G_a}$.

IF $\hat{Q}_6 = F_1 \wedge \hat{Q}_4$

then it may be canceled by the

GS mechanism.

E7/2

descent

$$\hat{\Omega}_5 = \mathcal{F}_1 \wedge \Omega_3$$

$$d\Omega_3 = \Omega_4, \quad d\mathcal{F}_1 = 0$$

$$d\hat{\Omega}_5 = \hat{\Omega}_6$$

$$\hat{H}_4 = \mathcal{F}_1 \wedge \hat{H}_2 = \mathcal{F}_1 \wedge \left(\sum_a c_a \Lambda^a dA^a\right).$$

$$\cancel{dH}\, \delta_\Lambda \Omega_3 = -dH_2$$

$$\delta_\Lambda \Omega_5 = -\mathcal{F}_1 \wedge dH_2 = -dH_4$$

GS mechanism:

Add a 2-field $B = B_{\mu\nu} dx^\mu \wedge dx^\nu$ to the theory

$$\mathcal{L}(\text{free } B) = \frac{1}{12} H^{\lambda\mu\nu} H_{\lambda\mu\nu}$$

$$H_{\lambda\mu\nu} = \partial_\lambda B_{\mu\nu} + \text{cyclic}$$

$$H = dB$$

Modify: $H = dB + \Omega_3$

under gauge transforms

$$B \to B' = B + H_2$$

$$B' = B + \sum_a c_a \Lambda^a dA^a$$

$$H' = dB + \cancel{dH_2} + \Omega_3 + \cancel{\delta\Omega_3}$$

$$= H \;:\; \text{gauge invariant.}$$

[7/3]

$$dH_{free} = ddB = 0$$

$$dH = d\omega_3 = Q_4$$

$$A_{dual}(H) = \int \Big(\underbrace{\tfrac{1}{2} H \wedge * H}_{\frac{1}{12} H^{\lambda\mu\nu} H_{\lambda\mu\nu} d^4x} + \frac{1}{16\pi^2} B \wedge F_1 \Big).$$

EoM for H

$$dH = Q_4 \rightsquigarrow \epsilon^{\alpha\lambda\mu\nu} \partial_\alpha H_{\lambda\mu\nu} = \epsilon F F$$

$$d * H = (\text{coeff})\, F_1$$

$$\hookrightarrow \partial^\lambda H_{\lambda\mu\nu} = (\text{coeff})\, \tilde{F}_{\mu\nu}$$

---

gauge transform: H is gauge inv.

but B is not

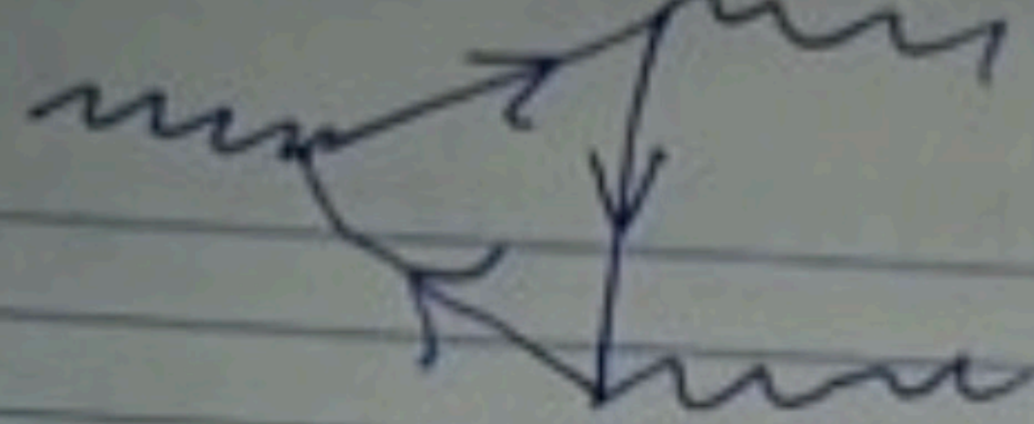
$$\Delta(B \wedge F_1) = F_1\, \theta \wedge \sum_a c_a \Lambda^a F^a = \hat{H}_4$$

$$\Delta_{gauge}(S) = \hat{H}_4 (\text{from } B \wedge F_1) - H_4 (\text{from anomaly})$$

$$= \text{cancel out.}$$

$\rightarrow$ net action gauge invariant.

E7/4)



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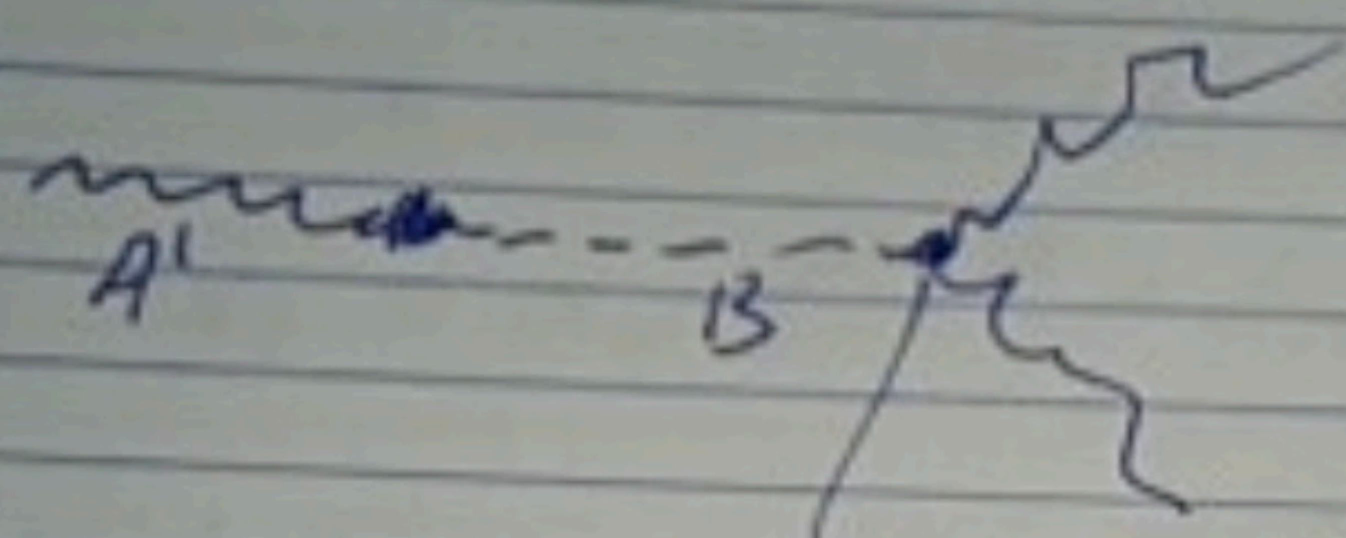


CS couplings.

Mult Right vertex by $K^{?}_{1\mu}$ (1^{st} ~)

→ get q_μ × something

Right vertex → another factor of q^μ

together: $q_\mu q^\mu = q^2$ cancels the propagator $\frac{i}{q^2+i\epsilon}$

→ altogether → local $K_{\mu}\epsilon^{\mu\nu\rho}$

→ cancels the Δ anomaly.

→ net $D^\mu J_\mu^a = 0$.

E7/5)

Combine gauge+gravity anomaly

$$\hat{Q}_6 = \mathrm{tr}_x(F\wedge F\wedge F) + \mathrm{tr}_x(F)\wedge \mathrm{tr}(R\wedge R).$$

→ If $\hat{Q}_6$ factorizes as $F_1 \wedge Q_4$

$$Q_4 = \mathrm{tr}(F\wedge F) + \mathrm{tr}(R\wedge R)$$

$$Q_4 = d\Omega_3^{\text{gauge}} + d\Omega_3^{\text{grav}}$$

Modify $H = dB + \Omega_3^{\text{gauge}} + \Omega_3^{\text{grav.}}$

$B\wedge F_1$ stays same.

→ gauge variance of B

$$\delta B = H_2^{\text{gauge}} + H_2^{\text{grav.}}$$

↳ $B\wedge F_1$ cancels the net anomaly

Anomaly in 6D.

$\hat{Q}_8$ is lin. comb of

$\mathrm{tr}(F\wedge F\wedge F\wedge F)$, $\mathrm{tr}(F\wedge F)\wedge \mathrm{tr}(F\wedge F)$,

$\mathrm{tr}(F\wedge F)\wedge \mathrm{tr}(R\wedge R)$

$\mathrm{tr}(R\wedge R\wedge R\wedge R)$; $\mathrm{tr}(R\wedge R)\wedge \mathrm{tr}(R\wedge R)$

traces over vector of $SO(5,1)$

$\boxed{E7/6}$ Suppose coeff $\mathrm{tr}(R\wedge R\wedge R\wedge R)$ is zero
ditto coeff $\mathrm{tr}(F\wedge F\wedge F\wedge F)$ is zero.

Also remaining $\hat{Q}_8$ factorizes as

$$\hat{Q}_8 = X_4 \wedge Y_4.$$

X_4 : lin comb of $\mathrm{tr}(F\wedge F)_{G_i}$
& $\mathrm{tr}(R\wedge R)$.

ditto Y_4.

$$X_4 = d\Omega_3^{tot} = d(\Omega_3^{gauge} + \Omega_3^{grav})$$

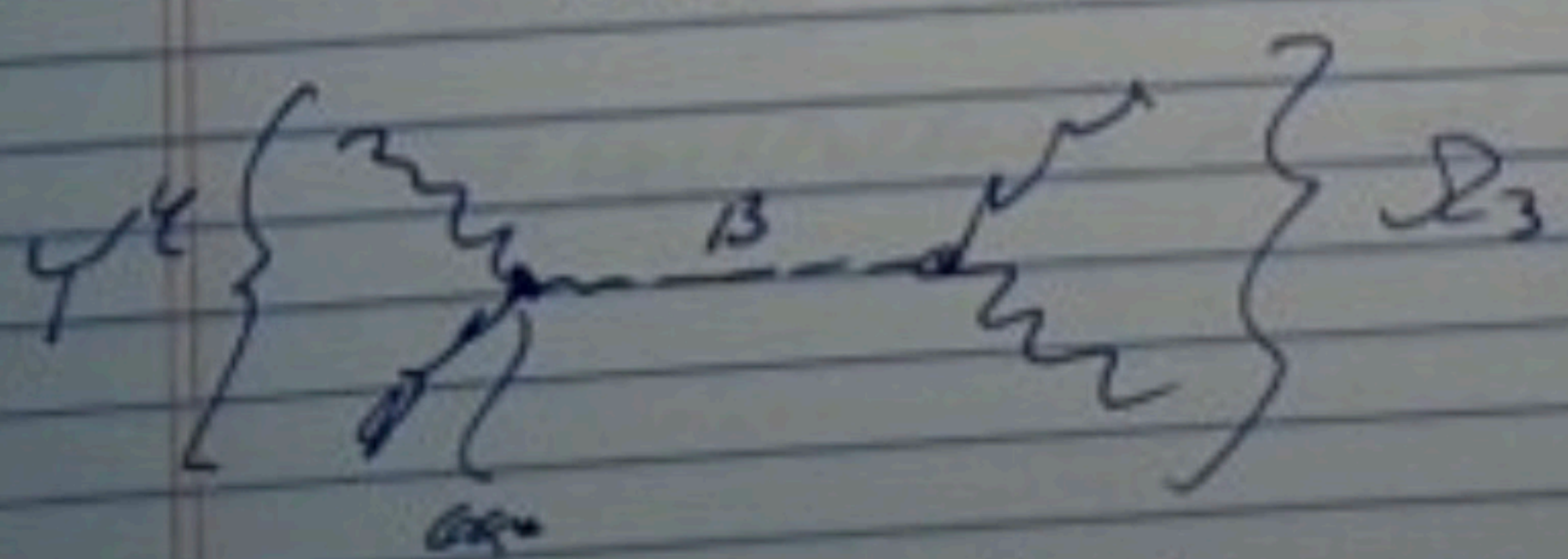
$$H = dB + \Omega_3^{gauge} + \Omega_3^{grav}.$$

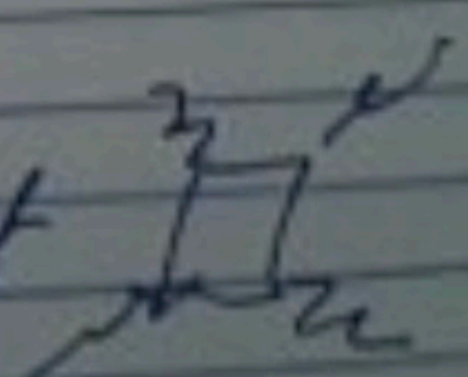
$$\delta_{gauge} B = H_2^{gauge} + H_2^{grav}$$

$\Rightarrow H$ is gauge invariant.

$$\mathrm{Action}_B = \int \left(\tfrac{1}{2} H\wedge * H + (\mathrm{coeff})\, B\wedge Y_4\right).$$

$$dH = X_4$$
$$d*H = Y_4$$



$\rightarrow$ cancels the anomaly of 

[7/7]

Anomaly descent

$\hat{Q}_8 = X_4 \wedge Y_4$, $dY_4 = 0$

$d\Omega_3 = X_4$

$\Omega_7 = \Omega_3 \wedge Y_4$

$d\Omega_7 = X_4 \wedge Y_4 = \hat{Q}_8$

gauge : $\delta\Omega_3 = -dH_2$
$\delta Y_4 = 0$

$\delta \Omega_7 = -dH_2 \wedge Y_4 = -d(H_2 \wedge Y_4)$

$\boxed{H_6 = H_2 \wedge Y_4}$

$\Delta S = (\text{coeff}) \int H_2 \wedge Y_4$

cancelled by $(\text{coeff}) \int (\delta B) \wedge Y_4$

→ GS anomaly cancellation.

→ can work only if the pure-gravity anomaly factorizes to $tr(R\wedge R) \wedge tr(R\wedge R)$.
→ Restriction on net # LH-RH Weyl fermions.

[E-7/8] $D=10$, original Green-Schwarz

Assume $\mathcal{N}=1$ SUSY.

fermions: 1 RH gravitino (MW)
1 LH (MW) fermion of grav sector

& 1 adjoint multiplet of LH MW fermions

$$\hat{Q}_{12} = \mathrm{tr}_{adj}(F\wedge F\wedge F\wedge F\wedge F\wedge F)$$
$$+(\text{coeff})\ \mathrm{tr}(R\wedge R)\wedge \mathrm{tr}_{adj}(F\wedge F\wedge F\wedge F)$$
$$+(\text{coeff})\ \mathrm{tr}(R\wedge R)^2\wedge \mathrm{tr}_{adj}(F\wedge F)$$
$$+(\text{coeff})\ \mathrm{tr}(R\wedge R\wedge R\wedge R)\wedge \mathrm{tr}_{adj}(F\wedge F).$$

+ pure gravity

$$a\times \mathrm{tr}(R^6)+b\,\mathrm{tr}(R^4)\wedge \mathrm{tr}(R^2)$$
$$+c\,\mathrm{tr}(R^2)^3$$

a,b,c dep on net size (G).
$=\#$ LH gauginos

GS mechanism requires factorization

$$\boxed{\hat{Q}_{12} = X_4\wedge Y_8}$$

In particular, coeff a of $\mathrm{tr}(R^6)$
must vanish

$\rightarrow$ Req size$(G)=496$.

E7/9

Also, the parasauge part of $\hat{Q}_{12}$ should not have $\mathrm{tr}(F^6)$ term but only $\mathrm{tr}(F^2)^3$ & $\mathrm{tr}(F^2)\wedge\mathrm{tr}(F^4)$.

G.S: For $SO(N)$ groups

$$\mathrm{tr}_{adj}(F^6) = (N-32)\times \mathrm{tr}_V(F^6) + 15\,\mathrm{tr}_V(F^2)\wedge\mathrm{tr}(F^4).$$

$\rightarrow$ $SO(32)$ is OK.

$\rightarrow$ For $SO(32)$

while $\hat{Q}_{12} = X_4 \wedge Y_8$

$$X_4 = \underset{\text{vector } \underline{32}}{\mathrm{tr}(F\wedge F)} - \underset{\text{covector } \underline{10}}{\mathrm{tr}(R\wedge R)}$$

$$H = dB + \Omega_3^{\text{gauge}} - \Omega_3^{\text{grav.}}$$

$$\underbrace{\mathcal{L} \supset \tfrac{1}{12}H^2 + B\wedge Y_8}$$

In 10 D, GS mechanism cancels all the anomalies for

$$G = SO(32) \text{ or } G = E_8\times E_8$$