Problem **1**(a):

The diagram (III) with the Higgs scalar in the s channel has two Yukawa vertices and a scalar propagator, thus

$$i\mathcal{M}_H \;=\; \frac{i}{s-M_H^2+iM_H\Gamma_H}\times\left[-iy_e\,\bar{v}(e^+)u(e^-)\right]\times\left[-iy_\mu\,\bar{u}(\mu^-)v(\mu^+)\right], \tag{S.1}$$

where y_e and y_μ are the respective Yukawa couplings of the electrons and the muons to the Higgs scalar. In the Standard Model, these same Yukawa couplings also determine the electron and the muon masses once the Higgs gets its VEV,

$$m_e \;=\; y_e\times\frac{v}{\sqrt{2}}\,,\quad m_\mu \;=\; y_\mu\times\frac{v}{\sqrt{2}}\,,\quad v\;\approx\;247\ \text{GeV}. \tag{S.2}$$

Since the muon and the electron — especially the electron — are rather light compared to the Higgs VEV, the Yukawa couplings

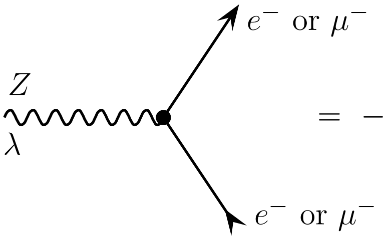
$$y_\mu \;=\; \sqrt{2}\frac{m_\mu}{v}\;\approx\;6.1\cdot10^{-4}\quad\text{and}\quad y_e\;=\;\sqrt{2}\frac{m_e}{v}\;\approx\;2.9\cdot10^{-6} \tag{S.3}$$

are rather small, much smaller than the gauge couplings $e\approx0.31$ or $(g'/4)\approx0.18$. Consequently, the Higgs-mediated amplitude $\mathcal{M}_H$ is almost 9 orders of magnitude weaker than the Z^0-mediated amplitude $\mathcal{M}_H$ and its effect is completely negligible; indeed, it's much smaller than the higher orders of perturbation theory in α_{QED} or α'_{SM}. Therefore, for our purposes we may approximate

$$\mathcal{M}_{\text{net}}\;\approx\;\mathcal{M}_\gamma\;+\;\mathcal{M}_Z\,. \tag{S.4}$$

Problem **1**(b):

The Zee and $Z\mu\mu$ vertices follow from the neutral weak current (3) and from the $SU(2)_W \times U(1)_Y$ quantum numbers of the the charged leptons e^- and μ^-: They both have $T^3 = -\frac{1}{2}$ and $Q = -1$, hence



$$= -ig'\gamma^\lambda\left(\tfrac{1}{4}\gamma^5 - \tfrac{1}{4} + \sin^2\theta_{\text{Weinberg}}\right) \approx -\frac{ig'}{4}\gamma^\lambda\gamma^5. \tag{S.5}$$

The Z propagator is spelled out in eq. (2), so altogether the amplitude of the diagram (II) evaluates to

$$i\mathcal{M}_Z = \frac{i\left(-g^{\lambda\nu} + \frac{q^\lambda q^\nu}{M_Z^2}\right)}{q^2 - M_Z^2 + iM_Z\Gamma_Z} \times \left[-\frac{ig'}{4}\,\bar{u}(\mu^-)\gamma_\lambda\gamma^5 v(\mu^+)\right] \times \left[-\frac{ig'}{4}\,\bar{v}(e^+)\gamma_\nu\gamma^5 u(e^-)\right] \tag{S.6}$$

where $q = p_- + p_+ = p'_- + p'_+$ and $q^2 = s$.

Problem **1**(c):

Proceeding similarly to the homework#10 (problem#2), we start by evaluating the electron-positron neutral current

$$\bar{v}\gamma_\nu\gamma_5\, u = v^\dagger\gamma^0\gamma_\nu\gamma^5 u = v^\dagger \begin{pmatrix} -\bar{\sigma}_\nu & 0 \\ 0 & +\sigma_\nu \end{pmatrix} u \tag{S.7}$$

for the ultra-relativistic electron and positron. As we saw back in homework#7, in the ultra-relativistic limit

$$\mathrm{u}(\mathrm{e}_\mathrm{L}^-) = \sqrt{2\mathrm{E}}\begin{pmatrix} \xi_L \\ 0 \end{pmatrix}, \quad \mathrm{u}(\mathrm{e}_\mathrm{R}^-) = \sqrt{2\mathrm{E}}\begin{pmatrix} 0 \\ \xi_R \end{pmatrix}, \quad \mathrm{v}(\mathrm{e}_\mathrm{L}^+) = -\sqrt{2\mathrm{E}}\begin{pmatrix} 0 \\ \eta_L \end{pmatrix}, \quad \mathrm{v}(\mathrm{e}_\mathrm{R}^+) = \sqrt{2\mathrm{E}}\begin{pmatrix} \eta_R \\ 0 \end{pmatrix}, \tag{S.8}$$

hence

$$\bar{v}(e_L^+)\gamma_\nu\gamma^5 u(e_L^-) = 0, \tag{S.9}$$

$$\bar{v}(e_L^+)\gamma_\nu\gamma^5 u(e_R^-) \;=\; -2E\,\eta_L^\dagger\sigma_\nu\xi_R\,, \tag{S.10}$$
$$\bar{v}(e_R^+)\gamma_\nu\gamma^5 u(e_L^-) \;=\; -2E\,\eta_R^\dagger\bar{\sigma}_\nu\xi_L\,, \tag{S.11}$$
$$\bar{v}(e_R^+)\gamma_\nu\gamma^5 u(e_R^-) \;=\; 0. \tag{S.12}$$

Specifically, for the electron and the positron colliding in the CM frame along the z axis,

$$\eta_L^\dagger\sigma_\nu\xi_R \;=\; (0,+1,-i,0)_\nu\,,\quad \eta_R^\dagger\bar{\sigma}_\nu\xi_L \;=\; (0,-1,-i,0)_\nu\,, \tag{S.13}$$

thus

$$\bar{v}(e_L^+)\gamma_\nu\gamma^5 u(e_R^-) \;=\; 2E(0,-1,+i,0)_\nu\,, \tag{S.14}$$
$$\bar{v}(e_R^+)\gamma_\nu\gamma^5 u(e_L^-) \;=\; 2E(0,+1,+i,0)_\nu\,, \tag{S.15}$$
$$\text{while } \bar{v}(e_L^+)\gamma_\nu\gamma^5 u(e_L^-) \;=\; \bar{v}(e_R^+)\gamma_\nu\gamma^5 u(e_R^-) \;=\; 0. \tag{S.16}$$

Note that similarly to QED pair production (diagram (I)), the electron and the positron must have opposite helicities, otherwise the amplitude vanishes.

Likewise, for the muon neutral current we have

$$\bar{u}(\mu_L^-)\gamma_\lambda\gamma^5 v(\mu_L^+) \;=\; \bar{u}(\mu_R^-)\gamma_\lambda\gamma^5 v(\mu_R^+) \;=\; 0$$

— which means that the μ^+ and the μ^- must have opposite helicities, — while

$$\bar{u}(\mu_R^-)\gamma_\lambda\gamma^5 v(\mu_L^+) \;=\; -2E\,\xi_R^\dagger\sigma_\lambda\eta_L\,, \tag{S.17}$$
$$\bar{u}(\mu_L^-)\gamma_\lambda\gamma^5 v(\mu_R^+) \;=\; -2E\,\xi_L^\dagger\bar{\sigma}_\lambda\eta_R\,. \tag{S.18}$$

For the muons scattered through angle θ away from the electron-positron axis in the zx plane, these formulae yield

$$\bar{u}(\mu_R^-)\gamma_\lambda\gamma^5 v(\mu_L^+) \;=\; 2E(0,-\cos\theta,-i,-\sin\theta)_\lambda\,, \tag{S.19}$$
$$\bar{u}(\mu_L^-)\gamma_\lambda\gamma^5 v(\mu_R^+) \;=\; 2E(0,+\cos\theta,-i,+\sin\theta)_\lambda\,. \tag{S.20}$$

Note that both the electron-positron and the muon neutral currents have no $\nu=0$ components in the CM frame, which makes them both transverse to q^ν. Consequently, in the amplitude (S.6) the

$$-g^{\lambda\nu} + \frac{q^\lambda q^\nu}{M_Z^2}$$

factor in the numerator of the Z^0 propagator can be reduced to just the $-g^{\lambda\nu}$ term. Thus

$$\mathcal{M}_Z = \frac{(g'/4)^2 g^{\nu\lambda}}{s - M_Z^2 + iM_Z\Gamma_Z} \times \bar{v}(E^+)\gamma_\nu\gamma^5 u(e^-) \times \bar{u}(\mu^-)\gamma_\lambda\gamma^5 v(\mu^+), \tag{S.21}$$

which evaluates to

$$\begin{aligned} \left\langle e_L^-, e_R^+ \right| \mathcal{M}_Z \left| \mu_L^-, \mu_R^+ \right\rangle &= \left\langle e_R^-, e_L^+ \right| \mathcal{M}_Z \left| \mu_R^-, \mu_L^+ \right\rangle \\ &= -\frac{(g'/4)^2 s}{s - M_Z^2 + iM_Z\Gamma_Z} \times (1+\cos\theta), \end{aligned} \tag{S.22}$$

$$\begin{aligned} \left\langle e_L^-, e_R^+ \right| \mathcal{M}_Z \left| \mu_R^-, \mu_L^+ \right\rangle &= \left\langle e_L^-, e_R^+ \right| \mathcal{M}_Z \left| \mu_R^-, \mu_L^+ \right\rangle \\ &= +\frac{(g'/4)^2 s}{s - M_Z^2 + iM_Z\Gamma_Z} \times (1-\cos\theta), \end{aligned} \tag{S.23}$$

while for all other helicity combinations $\mathcal{M}_Z = 0$.

Problem **1**(d):

The diagram (I) with the virtual photon in the s channel was evaluated in the homework#10. In the ultra-relativistic-fermions limit,

$$\left\langle e_L^-, e_R^+ \right| \mathcal{M}_\gamma \left| \mu_L^-, \mu_R^+ \right\rangle = \left\langle e_R^-, e_L^+ \right| \mathcal{M}_\gamma \left| \mu_R^-, \mu_L^+ \right\rangle = -e^2 \times (1+\cos\theta), \tag{S.24}$$

$$\left\langle e_L^-, e_R^+ \right| \mathcal{M}_\gamma \left| \mu_R^-, \mu_L^+ \right\rangle = \left\langle e_L^-, e_R^+ \right| \mathcal{M}_\gamma \left| \mu_R^-, \mu_L^+ \right\rangle = -e^2 \times (1-\cos\theta), \tag{S.25}$$

while for all other helicity combinations $\mathcal{M}_\gamma = 0$. Comparing these photon-mediated amplitudes to the Z^0-mediated amplitudes (S.22) and (S.23), we see exactly similar helicity dependence and angular dependence for both amplitudes. The only difference is the overall energy-dependent factor and the helicity-dependent overall sign,

$$\mathcal{M}_Z = \pm F(s)\times\mathcal{M}_\gamma, \tag{5}$$

$$F(s) = \left(\frac{g'/4}{e}\right)^2 \times \frac{s}{s-M_Z^2+iM_Z\Gamma_Z} \tag{6}$$

where in the Glashow–Weinberg–Salam theory

$$\frac{g'/4}{e} = \frac{1}{4\sin\theta_w\cos\theta_w} \to \frac{1}{\sqrt{3}} \quad \text{for } \sin^2\theta_w = \tfrac{1}{4}. \tag{S.26}$$

In light of eqs. (5),

$$\mathcal{M}_{\rm net} = (1\pm F(s))\times\mathcal{M}_\gamma\,, \tag{S.27}$$

specifically

$$\begin{aligned}\left\langle e_L^-,e_R^+\right|\mathcal{M}_{\rm net}\left|\mu_L^-,\mu_R^+\right\rangle &= \left\langle e_R^-,e_L^+\right|\mathcal{M}_\gamma\left|\mu_R^-,\mu_L^+\right\rangle\\ &= -e^2\times(1+F(s))\times(1+\cos\theta),\end{aligned} \tag{S.28}$$

$$\begin{aligned}\left\langle e_L^-,e_R^+\right|\mathcal{M}_{\rm net}\left|\mu_R^-,\mu_L^+\right\rangle &= \left\langle e_L^-,e_R^+\right|\mathcal{M}_\gamma\left|\mu_R^-,\mu_L^+\right\rangle\\ &= -e^2\times(1-F(s))\times(1-\cos\theta),\end{aligned} \tag{S.29}$$

while

$$\left\langle e_L^-,e_L^+\right|\mathcal{M}_{\rm net}\left|\mu_{\rm any}^-,\mu_{\rm any}^+\right\rangle = \left\langle e_R^-,e_R^+\right|\mathcal{M}_{\rm net}\left|\mu_{\rm any}^-,\mu_{\rm any}^+\right\rangle = 0, \tag{S.30}$$

$$\left\langle e_{\rm any}^-,e_{\rm any}^+\right|\mathcal{M}_{\rm net}\left|\mu_L^-,\mu_L^+\right\rangle = \left\langle e_{\rm any}^-,e_{\rm any}^+\right|\mathcal{M}_{\rm net}\left|\mu_R^-,\mu_R^+\right\rangle = 0. \tag{S.31}$$

The partial cross-sections (7) and (8) immediately follow from these amplitudes.

Problem **1**(e):

Let's start with the polarized partial cross-sections (8), whose angular dependence is either

$(1+\cos\theta)^2$ or $(1-\cos\theta)^2$. Integrating over the solid angle, we have

$$\int d^2\Omega\,(1\pm\cos\theta)^2 \;=\; 4\pi \;\pm\; 0 \;+\; \frac{4\pi}{3} \;=\; \frac{16\pi}{3}\,, \tag{S.32}$$

and also

$$\int\limits_{\theta<\frac{\pi}{2}} d^2\Omega\,(1\pm\cos\theta)^2 \;=\; 2\pi\int_0^1 d\cos\theta\,(1\pm\cos\theta)^2 \;=\; 2\pi(1\pm1+\tfrac{1}{3}) \;=\; \frac{(8\pm6)\pi}{3} \tag{S.33}$$

while

$$\int\limits_{\theta>\frac{\pi}{2}} d^2\Omega\,(1\pm\cos\theta)^2 \;=\; \frac{(8\mp6)\pi}{3}\,. \tag{S.34}$$

Consequently, the polarized cross-sections have forward-backward asymmetries $A=\pm\frac{3}{4}$.

Now consider the un-polarized cross-sections. Summing the partial cross-sections (7–8) over the muon spins and averaging over the electron's and positron spins, we get

$$\begin{aligned}\frac{d\sigma}{d\Omega} \;&=\; \frac{\alpha^2}{4s}\times\frac{1}{4}\Big(2\times|1+F(s)|^2(1+\cos\theta)^2 \;+\; 2\times|1-F(s)|^2(1-\cos\theta)^2 \;+\; 12\times0\Big)\\ &=\; \frac{\alpha^2}{8s}\Big(|1+F(s)|^2(1+\cos\theta)^2 \;+\; |1-F(s)|^2(1-\cos\theta)^2\Big).\end{aligned} \tag{S.35}$$

Integrating this partial cross-section over the forward and the backward hemispheres, we get

$$\sigma_{\text{forward}} \;=\; \frac{\pi\alpha^2}{12s}\times\Big(7\times|1+F(s)|^2 \;+\; |1-F(s)|^2\Big), \tag{S.36}$$

$$\sigma_{\text{backward}} \;=\; \frac{\pi\alpha^2}{12s}\times\Big(|1+F(s)|^2 \;+\; 7\times|1-F(s)|^2\Big), \tag{S.37}$$

and therefore the total cross-section

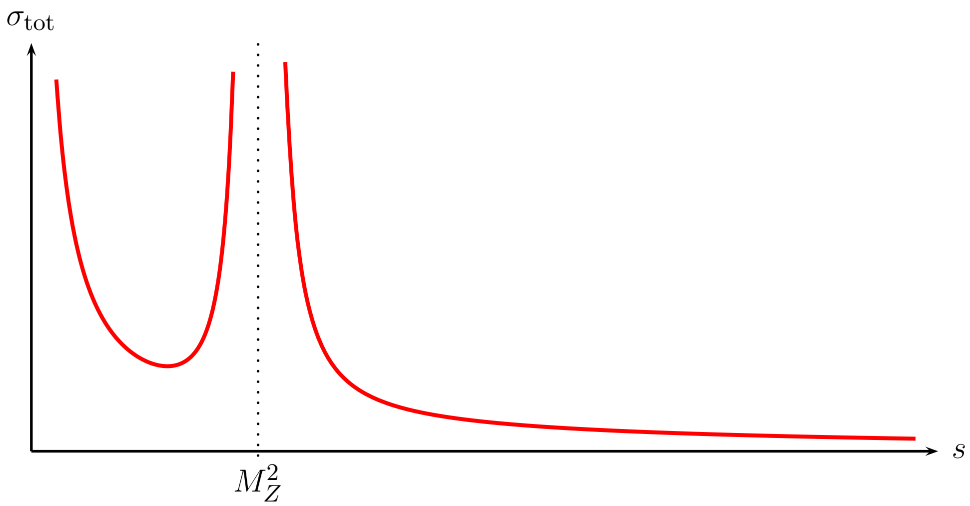
$$\sigma_{\text{total}} \;=\; \frac{2\pi\alpha^2}{3}\Big(|1+F|^2 \;+\; |1-F|^2\Big) \;=\; \frac{4\pi\alpha^2}{3}\Big(1 \;+\; |F(s)|^2\Big), \tag{S.38}$$

and the forward-backward asymmetry

$$A \;=\; \frac{6(|1+F|^2-|1-F|^2)}{8(|1+F|^2+|1-F|^2)} \;=\; \frac{3}{4}\times\frac{2\,\mathrm{Re}(F(s))}{1+|F(s)|^2}\,. \tag{S.39}$$

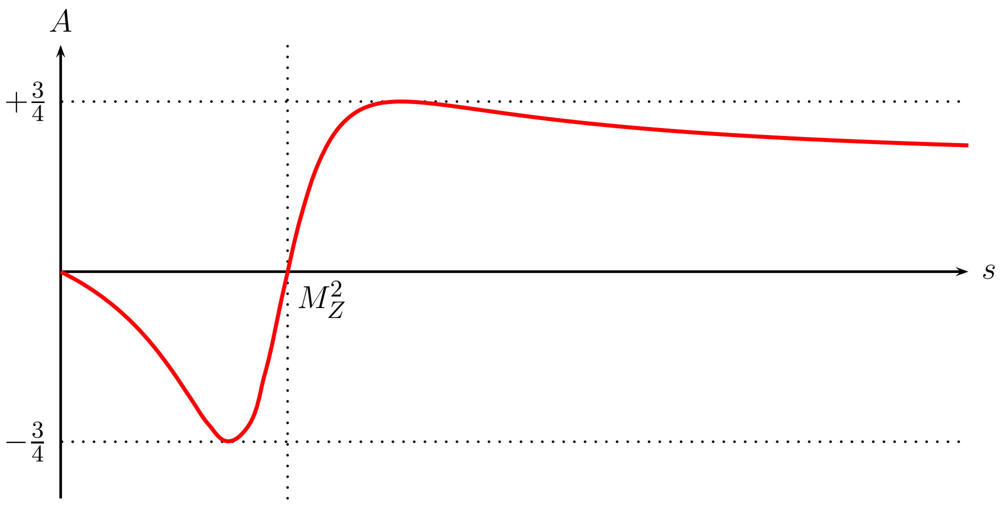
Specifically, for $F(s)$ as in eq. (6),

$$\sigma_{\text{total}} \;=\; \frac{4\pi\alpha^2}{3s}\left(1\;+\;\frac{1}{9}\,\frac{s^2}{(s-M_Z)^2+M_Z^2\Gamma_Z^2}\right) \tag{S.40}$$



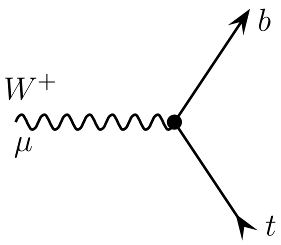
while

$$A \;=\; \frac{9}{2}\,\frac{s(s-M_Z^2)}{s^2+9(s-M_Z)^2+9M_Z^2\Gamma_Z^2} \;\approx\; \frac{9}{2}\,\frac{s(s-M_Z^2)}{s^2+9(s-M_Z)^2}\,. \tag{S.41}$$



Problem **2**(a):

The vertex connecting a top quark, a bottom quark, and a W boson obtains from eq. (10) as



$$= -\frac{ig}{2\sqrt{2}}\gamma^\mu(1-\gamma^5), \tag{S.42}$$

hence the top quark decay amplitude

$$\mathcal{M}(t \to b + W^+) \;=\; \mathcal{M}^\mu \mathcal{E}^*_\mu(k_W, \lambda_W) \quad \text{for} \quad \mathcal{M}^\mu \;=\; -\frac{g}{2\sqrt{2}} \times \bar{u}(b)\gamma^\mu(1-\gamma^5)u(t). \tag{S.43}$$

Problem **1**(b):

Let p be the initial top quark's momentum, p' the momentum of the final bottom quark, and $k = p - p'$ the momentum of the final W^+ boson. Then instead of the Ward identity $\mathcal{M}^\mu k_\mu = 0$ we have

$$\begin{aligned}
\bar{u}(b)\gamma^\mu(1-\gamma^5)u(t) \times k_\mu \;&=\; \bar{u}(b)\,\not{k}(1-\gamma^5)u(t) \\
&=\; -\bar{u}(b)\,\not{p}'(1-\gamma^5)u(t) \;+\; \bar{u}(b)\,\not{p}(1-\gamma^5)u(t) \\
&=\; -\bar{u}(b)\,\not{p}'(1-\gamma^5)u(t) \;+\; \bar{u}(b)(1+\gamma^5)\,\not{p}u(t) \\
&=\; -m_b\bar{u}(b) \times (1-\gamma^5)u(t) \;+\; \bar{u}(b)(1+\gamma^5) \times m_t u(t) \\
&=\; \bar{u}(b)[(m_t - m_b) + (m_t + m_b)\gamma^5]u(t) \;\neq\; 0.
\end{aligned} \tag{S.44}$$

and hence

$$\mathcal{M}^\mu k_\mu \;=\; -\frac{g}{2\sqrt{2}}\,\bar{u}(b)[(m_t - m_b) + (m_t + m_b)\gamma^5]u(t) \;\neq\; 0. \tag{S.45}$$

Problem $\mathbf{1}$(c):

For the massive vector particle W^μ at rest, $k^\mu = (m_W, \mathbf{0})$, its 3 polarization/spin states correspond to 3 orthonormal purely spatial vectors $\mathcal{E}^\mu(k,\lambda) = (0, \mathbf{e}_\lambda)$. Consequently,

$$\sum_\lambda \mathcal{E}^\mu(k,\lambda)\mathcal{E}^{\nu *}(k,\lambda) \;=\; \left\{ \begin{matrix} +\delta^{ij} & \text{for } \mu = i \text{ and } \nu = j, \\ 0 & \text{otherwise,} \end{matrix} \right\} \;=\; -g^{\mu\nu} \;+\; \frac{k^\mu k^\nu}{m_W^2}\,. \tag{S.46}$$

For a moving W^μ particle, the polarization vectors $\mathcal{E}^\mu(k,\lambda)$ obtain by Lorentz-boosting the $(0,\mathbf{e}_\lambda)^\mu$ 4-vector to the frame of the moving particle. Therefore, the polarization sum on the LHS of eq. (S.46) obtains by Lorentz boosting the tensor on the RHS of eq. (S.46), — which we may do by simply boosting the k^μ vector from $(m_w, \mathbf{0})$ to the momentum of the moving W^μ particle. And this is how we get eq. (11) for all on-shell momenta k^μ.

Finally, eq. (12) follows immediately from $\mathcal{M} = \mathcal{M}_\nu \mathcal{E}^{*\nu}$ and eq. (12). Indeed,

$$\begin{aligned} \sum_\lambda |\mathcal{M}|^2 \;&=\; \sum_\lambda \mathcal{M}_\nu \mathcal{E}^{\nu *} \times \mathcal{M}_\mu^* \mathcal{E}^\mu \;=\; \mathcal{M}_\nu \mathcal{M}_\mu^* \times \sum_\lambda \mathcal{E}^{\nu *}\mathcal{E}^\mu \\ &=\; \mathcal{M}_\nu \mathcal{M}_\mu^* \times \left(-g^{\mu\nu} \;+\; \frac{k^\mu k^\nu}{M_W^2} \right) \;=\; -\mathcal{M}^\mu \mathcal{M}_\mu^* \;+\; \frac{|\mathcal{M}^\mu k_\mu|^2}{M_W^2}\,. \end{aligned} \tag{S.47}$$

Problem $\mathbf{1}$(d):

For the top quark decay amplitude (S.43),

$$\mathcal{M}^{\mu *} \;=\; -\frac{g}{2\sqrt{2}}\,\bar{u}(t)\overline{\gamma^\mu(1-\gamma^5)}u(b), \tag{S.48}$$

where

$$\overline{\gamma^\mu(1-\gamma^5)} \;=\; \overline{(1-\gamma^5)}\,\overline{\gamma^\mu} \;=\; (1+\gamma^5)\gamma^\mu \;=\; \gamma^\mu(1-\gamma^5). \tag{S.49}$$

Consequently, summing over the final W particle's spin states, we get

$$\sum_\lambda |\mathcal{M}|^2 \;=\; \frac{g^2}{8} \times \bar{u}(b)\gamma^\mu(1-\gamma^5)u(b) \times \bar{u}(b)\gamma^\nu(1-\gamma^5)u(t) \times \left(-g_{\mu\nu} \;+\; \frac{k_\mu k_\nu}{M_W^2} \right). \tag{S.50}$$

Further summing over the b quark spins and averaging over the t quark spins, we arrive at

$$\begin{aligned}
\overline{|\mathcal{M}|^2} &= \frac{1}{2}\sum_{\lambda,s,s'}|\mathcal{M}|^2 \\
&= \frac{g^2}{8}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_W^2}\right)\times\frac{1}{2}\sum_{s,s'}\bar{u}(p',s')\gamma^\mu(1-\gamma^5)u(p,s)\times\bar{u}(p,s)\gamma^\nu(1-\gamma^5)u(p',s') \\
&= \frac{g^2}{8}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_W^2}\right)\times\frac{1}{2}\operatorname{tr}\Big((\not{p}'+m_b)\gamma^\mu(1-\gamma^5)(\not{p}+m_t)\gamma^\nu(1-\gamma^5)\Big).
\end{aligned} \tag{S.51}$$

Next, we need to evaluate the Dirac trace here. Opening the brackets $(\not{p}'+m_b)$ and $(\not{p}+m_t)$ and discarding the terms comprising odd numbers of Dirac matrices inside the trace, we get

$$\begin{aligned}
&\operatorname{tr}\Big((\not{p}'+m_b)\gamma^\mu(1-\gamma^5)(\not{p}+m_t)\gamma^\nu(1-\gamma^5)\Big) = \\
&\qquad = \operatorname{tr}\Big(\not{p}'\gamma^\mu(1-\gamma^5)\,\not{p}\gamma^\nu(1-\gamma^5)\Big) + m_b m_t \operatorname{tr}\Big(\gamma^\mu(1-\gamma^5)\gamma^\nu(1-\gamma^5)\Big) \\
&\qquad = \operatorname{tr}\Big(\not{p}'\gamma^\mu\,\not{p}\gamma^\nu(1-\gamma^5)^2\Big) + m_b m_t \operatorname{tr}\Big(\gamma^\mu\gamma^\nu(1+\gamma^5)(1-\gamma^5)\Big) \\
&\langle\!\langle \text{ using } (1-\gamma^5)^2 = 2(1-\gamma^5) \text{ and } (1+\gamma^5)(1-\gamma^5) = 0 \,\rangle\!\rangle \\
&\qquad = 2\operatorname{tr}\Big(\not{p}'\gamma^\mu\,\not{p}\gamma^\nu(1-\gamma^5)\Big) + m_b m_t \times 0 \\
&\qquad = 2\operatorname{tr}(\not{p}'\gamma^\mu\,\not{p}\gamma^\nu) - 2\operatorname{tr}(\not{p}'\gamma^\mu\,\not{p}\gamma^\nu\gamma^5) \\
&\qquad = 8p'^\mu p^\nu + 8p'^\nu p^\mu - 8(p'p)g^{\mu\nu} + 8i\epsilon^{\alpha\mu\beta\nu}p'_\alpha p_\beta\,.
\end{aligned}$$

Plugging this trace into eq. (S.51), we get

$$\begin{aligned}
\overline{|\mathcal{M}|^2} &= \frac{g^2}{8}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_W^2}\right)\times 4\Big(p'^\mu p^\nu + p'^\nu p^\mu - (p'p)g^{\mu\nu} + i\epsilon^{\alpha\mu\beta\nu}p'_\alpha p_\beta\Big) \\
&= \frac{g^2}{2}\left[\left(-(p'p) + \frac{(kp)(kp')}{m_W^2}\right)\times 2 - (pp')\times\left(-4 + \frac{k^2}{m_W^2} = -3\right) + 0\right] \\
&= g^2\left(\frac{(pp')}{2} + \frac{(kp)(kp')}{M_W^2}\right).
\end{aligned} \tag{S.52}$$

At this point, all that's left to do is kinematics. For a two-body decay $t \to b + W^+$, we have

$$\begin{aligned}
2(pp') &= p^2 + p'^2 - (p-p'=k)^2 = m_t^2 + m_b^2 - M_W^2\,, \\
2(pk) &= p^2 + k^2 - (p-k=p')^2 = m_t^2 + M_W^2 - m_b^2\,, \\
2(p'k) &= (p'+k=p)^2 - p'^2 - k^2 = m_t^2 - m_b^2 - M_W^2\,.
\end{aligned} \tag{S.53}$$

Consequently, (after a bit of algebra)

$$\begin{aligned}\overline{|\mathcal{M}|^2} &= \frac{g^2}{4M_W^2}\Big(M_W^2(m_t^2+m_b^2-M_W^2)\,+\,(m_t^2-m_b^2)^2\,-\,M_W^4\Big)\\ &= \frac{g^2}{4M_W^2}\Big((m_t^4+m_t^2M_W^2-2M_W^4)\,-\,m_b^2(2m_t^2-M_W^2)\,+\,m_b^4\Big),\end{aligned} \tag{S.54}$$

and since $m_b^2 \ll M_W^2, m_t^2$, we may neglect the bottom quark mass and approximate

$$\overline{|\mathcal{M}|^2} \;=\; \frac{g^2}{4M_W^2}\Big(m_t^4\,+\,m_t^2M_W^2\,-\,2M_W^4\Big). \tag{S.55}$$

Now let's calculate the decay rate of the top quark. As explained in my notes on phase space, for a 2-body decay like $t \to b + W^+$

$$\Gamma \;=\; \frac{|\mathbf{k}|}{8\pi m_t^2}\times\overline{|\mathcal{M}|^2} \tag{S.56}$$

where $\mathbf{k}$ is the 3-momentum of the W^+ (or minus the 3-momentum $\mathbf{p}'$ of the b quark) in the top quark's rest frame. In that frame $(pp') = m_t E_b$, hence by the first eq. (S.53)

$$E_b \;=\; \frac{2(pp')}{2m_t} \;=\; \frac{m_t^2+m_b^2-M_W^2}{2m_t} \;\approx\; \frac{m_t^2-M_W^2}{2m_t}\,, \tag{S.57}$$

and hence

$$|\mathbf{k}| \;=\; |\mathbf{p}'| \;\approx\; E_b \;\approx\; \frac{m_t^2-M_W^2}{2m_t}\,. \tag{S.58}$$

Plugging this value onto eq. (S.56), we finally arrive at

$$\Gamma \;=\; \frac{(m_t^2-M_W^2)}{16\pi m_t^3}\times\overline{|\mathcal{M}|^2} \;=\; \frac{g^2}{64\pi}\times\frac{(m_t^2-M_W^2)^2(m_t^2+2M_W^2)}{m_t^3M_W^2}\,, \tag{S.59}$$

or in terms of the Fermi constant G_F,

$$\Gamma \;=\; \frac{G_F m_t^3}{4\sqrt{2}\pi}\left(1-\frac{M_W^2}{m_t^2}\right)^2\left(1+2\frac{M_W^2}{m_t^2}\right). \tag{S.60}$$

Numerically, using $(g^2/4\pi) = 1/30$, $m_t = 173$ GeV, and $M_W = 80$ GeV, we get $\Gamma(t \to b+W^+) \approx 1.47$ GeV. Or rather, this is the net decay rate of the top quark into a W^+ and a b'

quark, which due to CKM mixing may turn out to be a b, an s, or a d quark. Experimentally, about 91% of top quark decays produces a bottom quark, the remaining 9% of decays yield a strange or down quark. Also, the experimental net width of the top quark is about 1.32 GeV, about 10% smaller than we have calculated. This discrepancy is due to QCD loop corrections which we did not take into account.