Problem **1**(a):

The non-abelian anomaly of a simple group like $SU(3)$ has general form

$$\mathcal{A}^{abc} = d^{abc}\times\mathcal{A}^{\text{net}} \quad\text{for } \mathcal{A}^{\text{net}} = A\binom{\text{multiplet}}{\text{of LHWF}} - A\binom{\text{multiplet}}{\text{of RHWF}} \tag{S.1}$$

where $A(\text{multiplet})$ is its cubic anomaly index. For the problem at hand, there are no RH Weyl fermions while the LH Weyl fermions comprise a reducible $\mathbf{6}+t\times\bar{\mathbf{3}}$ multiplet, thus

$$\mathcal{A}^{\text{net}} = A(\mathbf{6}) + 7\times A(\bar{\mathbf{3}}) = (N+4=7) + 7\times(-1) = 0. \tag{S.2}$$

In other words, the net $SU(3)^3$ anomaly cancels out.

Problem **1**(b):

The nonabelian $SU(7)$ flavor symmetry is automagically free from the $SU(3)$ anomaly,

$$\mathcal{A}^{abc} = 0 \quad\text{when } a,b\in SU(2),\quad c\in SU(7). \tag{S.3}$$

But the abelian $U(1)$ flavor symmetry is not protected by such magic, so let us check. For $a,b\in SU(3)$ while $c=Q\in U(1)$, we have

$$\mathcal{A}^{abc} = \sum_{\text{multiplets}} 2\,\text{tr}_{\text{colors}}(t^at^b)\times Q = \delta^{ab}\times\sum_{\text{multiplets}} 2R_2\times Q \tag{S.4}$$

where $R_2(\text{multiplet})$ is its quadratic index WRT $SU(3)$. For the problem at hand.

$$\begin{aligned}\sum_{(m)} 2R_2(m)Q(m) &= 2R_2(\mathbf{6})\times Q(\psi) + 2R_2(\bar{\mathbf{3}})\times Q(\chi)\times 7_{\text{flavors}}\\ &= 5\times(-7) + 1\times(+5)\times 7 = 0,\end{aligned} \tag{S.5}$$

so the $U(1)_Q$ flavor symmetry is also free from the $SU(3)$ anomaly.

Problem $\mathbf{1}$(c):

For the $SU(7)\times U(1)_Q$ flavor symmetry we have 3 types of allowed anomalies:

$$\begin{aligned}
\mathcal{A}^{abc} &= \mathcal{A}_{777}\times d^{abc}_{\mathrm{SU}(7)} \quad \text{for } a,b,c\in SU(7),\\
\mathcal{A}^{abc} &= \mathcal{A}_{771}\times \delta^{ab} \quad \text{for } a,b\in SU(7),\quad c=Q,\\
\mathcal{A}^{abc} &= \mathcal{A}_{111} \quad \text{for } a=b=c=Q,
\end{aligned}\tag{S.6}$$

and we should verify that for all 3 types the quarks $\psi^{ij}+\chi^f_i$ and the baryons $B^{ff'}$ have the same anomaly.

So let's start with the $\mathcal{A}_{777}$ anomaly. For the quarks,

$$\mathcal{A}_{777}[\text{quarks}] \;=\; 6\times A(\mathbf{1}) \;+\; 3\times A(\mathbf{7}) \;=\; 6\times 0 \;+\; 3\times 1 \;=\; +3,\tag{S.7}$$

while for the baryons

$$\mathcal{A}_{777}[\text{baryons}] \;=\; 1\times A(\mathbf{21}) \;=\; \big((N=7)-4\big) \;=\; +3.\tag{S.8}$$

Match!

Next, the $\mathcal{A}_{771}$ anomaly,

$$\mathcal{A}_{771} \;=\; \sum_{(m)} N(m)\times 2R_2(m)\times Q(m)\tag{S.9}$$

where R_2 is the $SU(7)$ quadratic index of a multiplet (m) and $N(m)$ is the number of its copies. For the quarks,

$$\begin{aligned}
\mathcal{A}_{771}[\text{quarks}] &= 6\times 2R_2(\mathbf{1})\times Q(\psi) \;+\; 3\times 2R_2(\mathbf{7})\times Q(\chi)\\
&= 6\times 0\times(-7) \;+\; 3\times 1\times(+5) \;=\; +15,
\end{aligned}\tag{S.10}$$

while for the baryons

$$\mathcal{A}_{771}[\text{baryons}] \;=\; 1\times R_2(\mathbf{21})\times Q(B) \;=\; 1\times 5\times(+3) \;=\; +15.\tag{S.11}$$

Match!

Finally, the $\mathcal{A}_{111}$ anomaly

$$\tfrac{1}{2}\mathcal{A}_{111} \;=\; \mathrm{tr}(Q^3) \;=\; \sum_{\text{fermions}} N \times Q^3. \tag{S.12}$$

For the quarks

$$\tfrac{1}{2}\mathcal{A}_{111}[\text{quarks}] \;=\; 6\times Q^3(\psi) \;+\; 21\times Q^3(\chi) \;=\; 6\times(-7)^3 \;+\; 21\times(+5)^3 \;=\; +567, \tag{S.13}$$

while for the baryons

$$\tfrac{1}{2}\mathcal{A}_{111}[\text{baryons}] \;=\; 21\times(+3)^3 \;=\; +567. \tag{S.14}$$

Match!

For completeness sake, let me also verify the gravitational anomaly of the $U(1)$ charge Q:

$$\mathrm{tr}_{\text{quarks}}(Q) \;=\; 6\times(-7) \;+\; 21\times(+5) \;=\; +63, \tag{S.15}$$

while

$$\mathrm{tr}_{\text{baryons}}(Q) \;=\; 21\times(+3) \;=\; +63. \tag{S.16}$$

Match!