15/1

n(water)

Low freq. $\varepsilon \simeq 81$, $\mu = 1$ $n = \sqrt{\varepsilon\mu} \simeq 9$

~~Op~~ Vis. light $n \simeq 1.33$

For IR n is complex

$\rightarrow$ water is not transparent.

Plane wave $\vec{E} = \vec{\mathcal{E}}_0 e^{ikz - i\omega t}$

$\vec{H} = \mathcal{H}_0 e^{ikz - i\omega t}$

$$K = \frac{n(\omega)\omega}{c}$$

complex $n(\omega)$, $\mathrm{Im}\, n > 0$

K is complex $K = K_r + i\varkappa$

$$\vec{E}, \vec{H} \sim e^{ik_r z - \varkappa z - i\omega t}$$

$$\text{Intensity} = S_z = \frac{1}{2}\mathrm{Re}(\vec{E}^* \times \vec{H}) = S_0 e^{-2\varkappa z}$$

$\rightarrow$ the wave attenuates @ rate $2\varkappa$

$$\text{penetration depth} = \frac{1}{2\varkappa} = \frac{c}{2\omega} \times \frac{1}{\mathrm{Im}\, n(\omega)}$$

$$= \frac{\lambda/4\pi}{\mathrm{Im}\, n(\omega)}$$

(15/2)

Lost EM power → heat.

$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \varepsilon \varepsilon_0 \vec{E}$

suppose $\vec{E} = E_0 e^{-i\omega t}$

but P lags behind E in time

$$\vec{P} = P_0 e^{-i\omega(t-\delta t)}$$

$$\vec{D} = \underbrace{(\varepsilon_0 \vec{E}_0 + P_0 e^{+i\omega\delta t})}_{\varepsilon_0 \varepsilon} e^{-i\omega t}$$

complex ε, $\mathrm{Im}\,\varepsilon > 0$

work of El. field

$$\delta W = \iiint d^3x \, \vec{E} \cdot \delta \vec{D}$$

Power density $= \boxed{\vec{E} \cdot \frac{\partial \vec{D}}{\partial t}}$

$$\vec{E} = E_0 e^{-i\omega t}, \quad \vec{D} = \varepsilon \varepsilon_0 E_0 e^{-i\omega t}$$

time-avgd power density

$$\left\langle \frac{dP}{dvol} \right\rangle = \frac{1}{2} \mathrm{Re}\left(\vec{E}^* \cdot \frac{\partial \vec{D}}{\partial t}\right)$$

$$= \frac{1}{2} \mathrm{Re}\left(\vec{E}^* \cdot -i\omega \varepsilon \varepsilon_0 \vec{E}\right)$$

$$= \frac{\varepsilon_0}{2} |\vec{E}|^2 \cdot \mathrm{Re}(-i\omega\varepsilon)$$

$$= \frac{\varepsilon_0}{2} |E|^2 \cdot \boxed{\omega \, \mathrm{Im}\, \varepsilon(\omega)}$$

15/3

Im ϵ causes dissipation

Conductivity also causes dissipation

$$\vec{J} = \sigma \vec{E}$$

$$\text{Power density} = \vec{E}^* \cdot \vec{J} = \sigma \bar{E}^2$$

$$\text{time-avg'd} \rightarrow \frac{1}{2} Re(\vec{E}^* \cdot \vec{J}) = \frac{|\vec{E}|^2}{2} \cdot Re(\sigma)$$

Altogether: dissipated power density

$$\frac{P}{vol} = \frac{|\vec{E}|^2}{2} Re(\underbrace{\sigma - i\epsilon\epsilon_0\omega}_{})$$

$$\boxed{\sigma_c = \sigma(\omega) - i\epsilon_0 \epsilon(\omega)\omega}$$

is called complex conductivity.

Same $\sigma_c(\omega)$ appears in

$$\nabla \times \vec{H} = \vec{J} + \vec{J}_{disp} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \sigma_c \vec{E}$$

$$\epsilon_{eff}(\omega) = \frac{i\sigma_c(\omega)}{\epsilon_0 \omega} = \frac{i\sigma(\omega)}{\epsilon_0 \omega} + \epsilon(\omega).$$

For $\mu = 1$, $n = \sqrt{\epsilon_{eff}}$

complex $\epsilon_{eff} \rightarrow$ complex n.

$\rightarrow$ attenuation

15/4)

$$\frac{\text{Power loss}}{\text{vol}} = \frac{\epsilon_0 |\vec{E}|^2}{2} \omega \, \text{Im}\, \epsilon_{\text{eff}}(\omega)$$

$$\text{Intensity} = \frac{\text{Power}}{\text{surface}} = \frac{1}{2}\text{Re}(\vec{E}^* \times \vec{H})$$

$$= \frac{|\vec{E}|^2}{2}\text{Re}\left(\frac{1}{Z}\right) = \frac{|\vec{E}|^2 \epsilon_0}{2} c \,\text{Re}\sqrt{\epsilon_{\text{eff}}}$$

$$\frac{1}{Z} = \frac{\sqrt{\epsilon_{\text{eff}}}}{\epsilon_0 c \sqrt{\epsilon_{\text{eff}}}}$$

$$\epsilon_{\text{eff}} = n^2.$$

$$= \frac{\text{Power loss density}}{\text{Intensity}} = \frac{\omega}{c}\frac{\text{Im}(n^2)}{\text{Re}(n)}$$

attenuation rate $= 2\kappa$.

$$\frac{\text{Im}(n^2)}{\text{Re}(n)} = 2\,\text{Im}(n).$$

$$2\kappa = \frac{2\omega}{c}\text{Im}(n).$$

agrees w. $k_r + i\kappa = \frac{\omega}{c} n.$

In magnetic materials, $\mu \neq 1$

$\rightarrow$ extra power $\iiint d^3x \; \vec{H} \cdot \frac{\partial \vec{B}}{\partial t}$

$$\rightarrow \frac{\text{extra power loss}}{\text{vol.}} = \frac{\mu_0}{2}|\vec{H}|^2 \times \omega\, \text{Im}(\mu(\omega)).$$

$\mathrm{Im}(\mu(\omega))$ also causes power loss

→ Attenuation via

~~via~~ $\mathrm{Im}(n = \sqrt{\epsilon_{eff}\,\mu})$