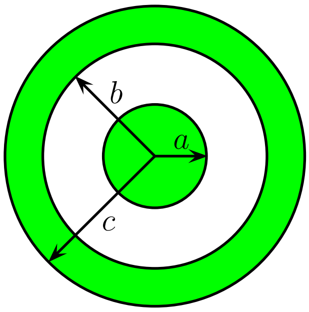
Please do not waste your time re-deriving any textbook formulae, or any formulae from my notes or homework solutions, just write them down and use them.

1. [15 points] A metal ball of radius a is surrounded by a concentric metal shell (inner radius b, outer radius c). The ball has net charge Q_1, the shell has net charge Q_2.



   (a) [5 pt] Write down the electric field $\mathbf{E}(\mathbf{r})$ and the potential $V(\mathbf{r})$ at all locations in this system. Take $V(\infty) = 0$.

   (b) [5 pt] Find the surface charge densities at $r = a$, $r = b$, and $r = c$.

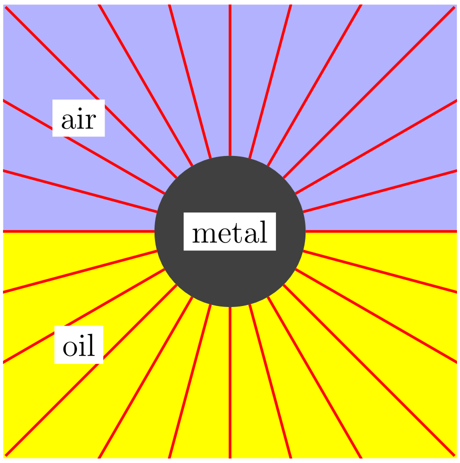
   (c) [5 pt] If the outer shell becomes grounded — that is, connected by a wire to the ground with $V_{\text{ground}} = V(\infty) = 0$, — what would happen to the charge of the shell? How would the new charge Q_2' be distributed between the inner surface, outer surface, and the bulk volume of the metal shell?

2. [16 points] Two point charges Q_1 and Q_2 are held at the same height h above an infinite horizontal conducting plane, and at distance d from each other.

   (a) [8 pt] Find the net force on the charge Q_1. (An answer in components is OK, don't waste your time converting it to magnitude and direction.)

   (b) [8 pt] Find the net electrostatic energy of the system.

3. [21 points] A charged metal sphere or radius R floats equator-deep in a pool of oil, which is a linear dielectric with susceptibility χ. The upper half of the sphere sticks out in the air (which you may approximate as vacuum). Despite the asymmetry between the oil and the air, the electric field outside the metal is spherically symmetric,



$$\mathbf{E} = \frac{k}{r^2}\,\hat{\mathbf{r}} \tag{1}$$

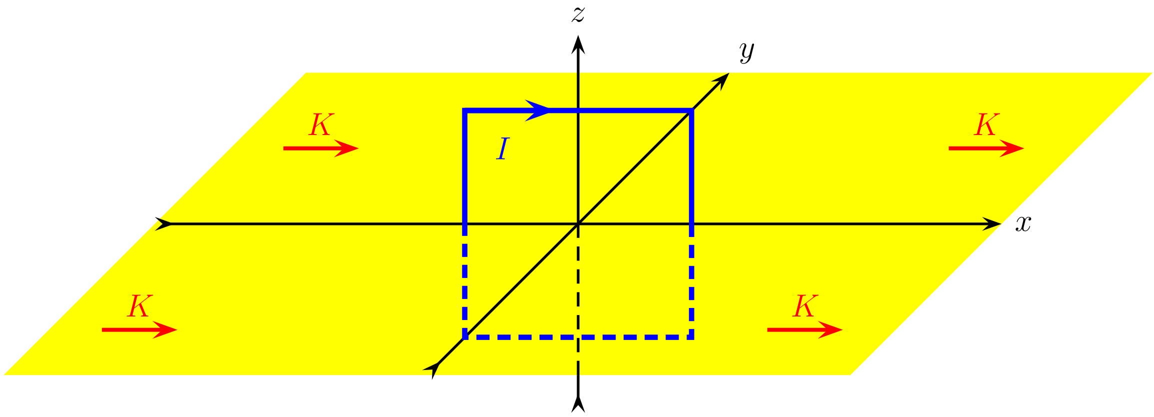
for some constant k.

(a) [7 pt] Verify that this electric field obeys the boundary conditions at the oil-air interface. Hint: this interface lies in the same plane as the sphere's center.

(b) [7 pt] Find all the free charge densities and the bound charge densities in the system. (Both surface and volume densities, if any.)

(c) [7 pt] If the metal sphere is used as a capacitor plate (with the other "plate" at the infinity), what is the capacitance of this capacitor?

4. [18 points] A current sheet fills up the xy plane with uniform surface current density $\mathbf{K}$ in the $+x$ direction. A square $a \times a$ loop in the xz plane carrying current I straddles the sheet:



(a) [6 pt] Write down the magnetic field of the current sheet above and below the sheet.

(b) [6 pt] Should there be a net magnetic force on the square loop? If yes, what should be its direction? (No calculations in this part, just qualitative arguments.)

(c) [6 pt] Calculate the net force on the square loop.

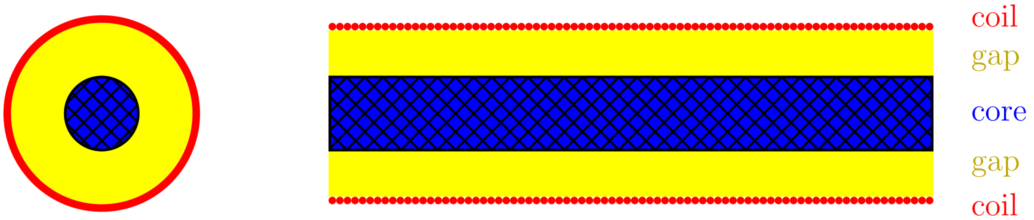
5. [15 points] A small permanent magnet of net magnetic dipole moment $\mathbf{m}$ is placed at the center of a circular wire ring of radius R. Allow for a generic angle α between the direction of $\mathbf{m}$ and the plane of the ring.

(a) [10 pt] Use the dipole's vector potential $\mathbf{A}$ to show that the magnetic flux through the ring is

$$\Phi = \frac{\mu_0|\mathbf{m}|}{2R}\sin\alpha. \tag{2}$$

(b) [5 pt] Let's rotate the magnet with angular velocity ω so that $\alpha(t) = \omega t$. Find the EMF induced in the wire ring.

6. [15 points] A solenoid coil has N loops, length ℓ, and radius $b \ll \ell$. An iron core of relative permeability μ is placed inside the solenoid; the core has the same length ℓ as the solenoid but a smaller radius $a < b$. The gap between the core and the coil is filled with a non-magnetic material.



(a) [10 pt] Steady current I flows through the solenoid coil. Find the magnetic fields $\mathbf{B}$ and $\mathbf{H}$ inside the iron core and also in the gap between the core and the coil.

(b) [5 pt] Find the self-inductance of the coil.