⋆ Please do not waste your time copying parts of the textbook, the class notes, or the homework solutions, or re-deriving formulae I have derived in class. Simply write down the equations you need and apply them to the problem at hand.

1. [25 points] Electric charge $+Q$ is uniformly distributed along a circular ring of radius R, while a point charge $-Q$ sits at the center of the ring. Consider the electrostatic potential $V(r, \theta, \phi)$ of this charge system and the multipole expansion of this potential at long distances $r \gg R$.
   (a) [10 pt] What is the leading multipole of this expansion? That is, what is the lowest–ℓ multipole with a non-zero multipole moment?
   (b) [10 pt] Calculate the leading multipole moment. Use the axial symmetry of the system in question.
   (c) [5pt] Write down the potential $V(r, \theta, \phi)$ at large $r \gg R$ in the leading multipole approximation.

2. [20 points] A capacitor is made of two coaxial tubes of radii a (the inner tube) and b (the outer tube) and length $L \gg b > a$. The dielectric filling the space between the tubes has non-uniform dielectric constant

$$\epsilon(s, \phi, z) = \epsilon(s \text{ only}) = \frac{b^2}{s^2}. \tag{1}$$

   The capacitor is charged: the inner tube has free charge $+Q$ while the outer tube has free charge $-Q$.
   (a) [10 pt] Find the electric displacement field $\mathbf{D}$, the electric tension field $\mathbf{E}$, and the dielectric polarization $\mathbf{P}$ between the two tubes.
   (b) [10 pt] Find the voltage between the tubes, the capacitance of the capacitor, and the net electrostatic energy of the system.
   (⋆) For extra credit [10 points]:
   Find all the bound charges in the volume and on the surfaces of the dielectric and verify that the net bound charge is zero.

3. [30 points] Consider two infinitely long straight wires that are **not parallel** to each other. Specifically, wire#1 with current I_1 runs along the z axis while wire#2 with current I_2 spans

$$\mathbf{r} \;=\; a\,\hat{\mathbf{x}} \;+\; \ell\sin\theta\,\hat{\mathbf{y}} \;+\; \ell\cos\theta\,\hat{\mathbf{z}}\,, \quad \text{fixed } a,\theta, \quad \text{variable } \ell \text{ from } -\infty \text{ to } +\infty. \tag{2}$$

(a) [15 pt] Write down the magnetic field $\mathbf{B}_1(x,y,z)$ of the first wire and calculate the force $d\mathbf{F}$ it exerts on the element $d\ell$ of the second wire.

(b) [15 pt] Calculate the net force between the two wires. Note: the *net* force rather than the force per unit length.

Here are some formulae to help your calculations:

$$\frac{\hat{\phi}}{s} \;=\; \frac{x\,\hat{\mathbf{y}} \,-\, y\,\hat{\mathbf{x}}}{x^2+y^2}\,, \qquad \int\limits_{-\infty}^{+\infty} \frac{d\ell}{a^2 \,+\, c^2\ell^2} \;=\; \frac{\pi}{ac}\,, \qquad \int\limits_{-\infty}^{+\infty} \frac{\ell\,d\ell}{a^2 \,+\, c^2\ell^2} \;=\; 0. \tag{3}$$

4. [25 points] Find the magnetic field of a helical current density

$$\mathbf{J}(x,y,z) \;=\; J_0\Big(\cos(kz)\,\hat{\mathbf{x}} + \sin(kz)\,\hat{\mathbf{y}}\Big) \qquad \langle\!\langle\, \text{constant } J_0,\ \text{constant } k\,\rangle\!\rangle \tag{4}$$

flowing through the whole space. Note several symmetries of this current, including invariance under simultaneous translations in z direction and rotations around the z axis.

(a) [6 pt] Can this current be steady?

(b) [7 pt] Without taking any integrals or solving any differential equations, use the symmetries of the current density (4) to argue that the vector potential should have form

$$\mathbf{A}(x,y,z) \;=\; A_0\Big(\cos(kz)\,\hat{\mathbf{x}} \,+\, \sin(kz)\,\hat{\mathbf{y}}\Big) \tag{5}$$

for some constant A_0.

(c) [7 pt] Check that the vector potential (5) obeys the Poisson equation $\nabla^2\mathbf{A} = -\mu_0\mathbf{J}$ for the appropriate value of A_0 and find that value.

(d) [5 pt] Finally, write down the magnetic field $\mathbf{B}$ for the vector potential (5).