

1. First of all, refresh your memory of basic SUSY. And if you have not finished reading about it during the summer, please do it now.

Warning: to follow this class, you absolutely have to be familiar with at least the first 3 items on the required self-study list: (1) The SUSY algebra, the supermultiplets, and the $\mathcal{N} = 1$ superspace in 4 dimensions; (2) The chiral superfields and their interactions, the superpotential, and the Kähler function. (3) The abelian vector superfields and SQED. The remaining items can wait a bit, but these three are needed from day one: If you do not grok the superfield formalism, you are SOL.

2. Second, an exercise in D -operator algebra.

(a) Calculate the commutators $[D_\alpha, \bar{D}^2]$, $[\bar{D}_{\dot{\alpha}}, D^2]$, and $[D^2, \bar{D}^2]$.

(b) Show that $D^\alpha \bar{D}^2 D_\alpha = \bar{D}_{\dot{\alpha}} D^2 \bar{D}^{\dot{\alpha}}$.

(c) Show that $2D^\alpha \bar{D}^2 D_\alpha - D^2 \bar{D}^2 - \bar{D}^2 D^2 = 16\partial^2$.

(d) Now, let's define 3 projector operators,

$$\Pi_C = \frac{-1}{16\partial^2} \bar{D}^2 D^2, \quad \Pi_A = \frac{-1}{16\partial^2} D^2 \bar{D}^2, \quad \Pi_L = \frac{+1}{8\partial^2} D^\alpha \bar{D}^2 D_\alpha. \quad (1)$$

Verify that they are indeed projectors, *i.e.* satisfy $\Pi_C^2 = \Pi_C$, $\Pi_A^2 = \Pi_A$, and $\Pi_L^2 = \Pi_L$. Also, verify that they all commute with each other and add up to 1.

Hint: Use $DDD = 0$ (for any indices of the 3 D operators) and likewise $\bar{D}\bar{D}\bar{D} = 0$.

These 3 projector operators correspond to the three kinds of linearly-constrained superfields: A chiral superfield Φ satisfies $\bar{D}_{\dot{\alpha}}\Phi = 0$, and anti-chiral superfield $\bar{\Phi}$ satisfies $D_\alpha\bar{\Phi} = 0$, and a linear superfield L satisfies $D^2L = \bar{D}^2L = 0$.

- (e) Verify the correspondence: show that for any superfield $X(x, \theta, \bar{\theta})$, $\Pi_C X$ is chiral, $\Pi_A X$ is anti-chiral, $\Pi_L X$ is linear, and also that $\Pi_C \Phi = \Phi$ for any chiral superfield Φ , $\Pi_A \bar{\Phi} = \bar{\Phi}$ for any anti-chiral $\bar{\Phi}$, and $\Pi_L L = L$ for any linear superfield L .

3. Finally, consider a free massive vector superfield: a real general superfield $V(x, \theta, \bar{\theta})$ with action

$$S = \int d^4x d^2\theta d^2\bar{\theta} V(m^2 + \frac{1}{8}D^\alpha \bar{D}^2 D_\alpha)V. \quad (2)$$

Off-shell, $V(x, \theta, \bar{\theta})$ is not subject to any constraints except $V^\dagger = V$, but on-shell it satisfies the equation of motion

$$(m^2 + \frac{1}{8}D^\alpha \bar{D}^2 D_\alpha)V = 0. \quad (3)$$

- (a) Show that eq. (3) implies the Klein–Gordon equation $(m^2 + \partial^2)V = 0$ as well as $D^2V = \bar{D}^2V = 0$. Consequently, *on-shell* V is a linear superfield.
- (b) Expand $V(x, \theta, \bar{\theta})$ into component fields. Focus on the bosonic components C , f , f^* , A^μ , and \mathcal{D} and show that $D^2V = \bar{D}^2V = 0$ implies $f = f^* = 0$, $\mathcal{D} = -\partial^2 C$ (and hence $\mathcal{D} = m^2 C$), and $\partial_\mu A^\mu = 0$. Consequently, the independent bosonic fields comprise a massive vector field $A^\mu(x)$ and a single real scalar $C(x)$.
- (c) Now focus on the fermionic component fields χ_α , λ_α and their conjugates $\bar{\chi}_{\dot{\alpha}}$ and $\bar{\lambda}_{\dot{\alpha}}$ and show that they satisfy the Weyl equations. This is equivalent to a Dirac spinor field Ψ and its conjugate $\bar{\Psi}$, each satisfying the appropriate Dirac equation.