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Banks-Zaks in ordinary QCD

$$\beta(g) = \frac{b_1}{16\pi^2} g^3 + \frac{b_2}{(16\pi^2)^2} g^5 + \cdots$$

$$b_1 = -\frac{11}{3} N_c + \frac{2}{3} N_f$$

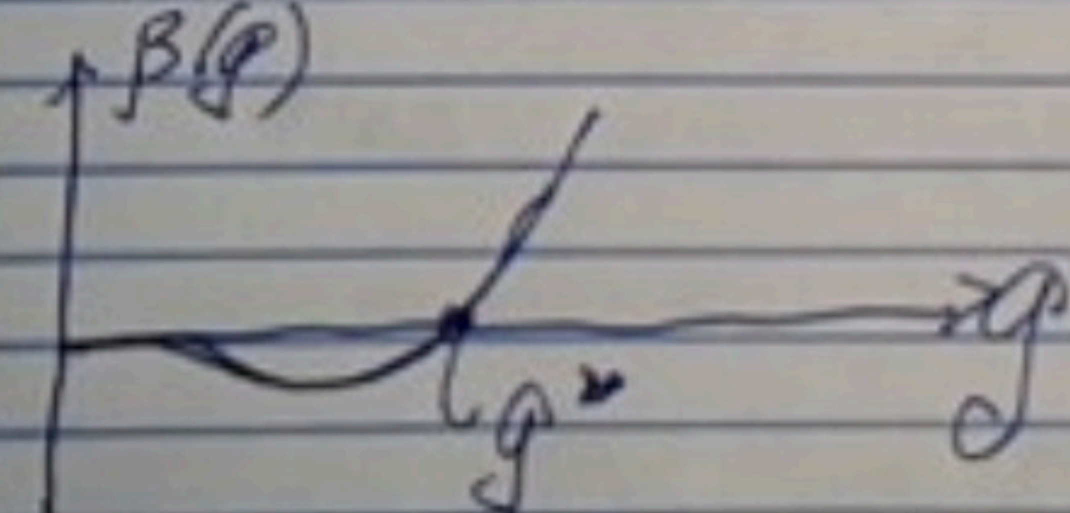
Limit of large N_c, large N_f

$$\frac{N_f}{N_c} = \frac{11}{2} - \text{small } \epsilon$$

Assume all quark flavors are massless
$\Rightarrow$ follow RG flow to deep IR.

$$b_1 = -\frac{2}{3} N_c \epsilon < 0$$

but $b_2 > 0$, $b_2 = O(N_c^2)$



$$\frac{g^{*2} N_c}{16\pi^2} = O(\epsilon) \Rightarrow \text{small enough}$$
to trust 2-loop pert-theory

g^*: IR-attractive fixed point

$\Rightarrow g(E)$ flows to g^* in deep IR.

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Banks-Zaks: conformal regime of QCD

3 generic IR regimes of QCD

1) $\frac{n_f}{n_c} \geq \frac{11}{2} \to$ no AF, IR weak
non-abelian Coulomb phase

2) $\boxed{\frac{11}{2} > \frac{n_f}{n_c} > \overline{X}}$ conformal window.
conformal phase
conf. invariance in deep IR.

3) $\frac{n_f}{n_c} < \overline{X}$
confining phase:
quark confinement, χSB.
no massless particles except
Goldstone pseudoscalars

lattice "experiments": $\overline{X} \sim 3$ to 4

8/3 ~~SQ~~ <u>SQCD</u>

$$\beta(g) = \frac{g^3}{16\pi^2 - 2N_c g^2}\left[-3N_c + N_f(1-2\gamma)\right]$$

~~g like~~ β vanishes when ~~$\gamma = -3N_c$~~

$$\gamma = \gamma^* = \frac{N_f - 3N_c}{2N_f}$$

For $\frac{N_f}{N_c} = 3 - \epsilon \Rightarrow \gamma^* \approx -\frac{\epsilon}{6}$

$$\gamma_{\text{1 loop}} = -\frac{g^2 C(\text{fund})}{8\pi^2} \qquad C(\text{fund}) = \frac{N_c^2-1}{2N_c} \approx \frac{N_c}{2}$$

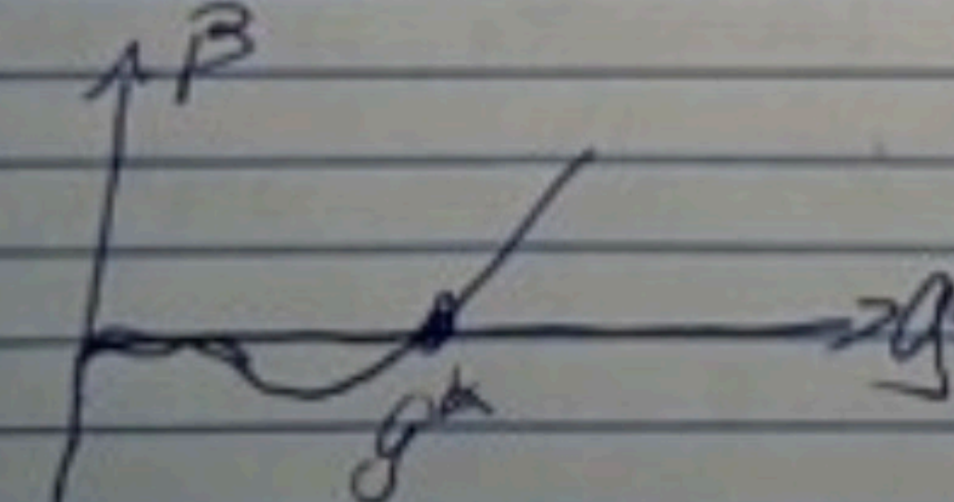
For small ϵ, $\gamma(g) = \gamma^*$

at weak coupling $\frac{g^2 N_c}{8\pi^2} \approx \frac{\epsilon}{3}$

$\rightarrow$ can trust pert. theory

$$\beta(g) \approx \frac{g^3}{16\pi^2}\left[-\epsilon - 6\gamma(g)\right]$$

$\Rightarrow$



$\Rightarrow$ Banks Zaks IR attractor.

deep IR $\rightarrow$ conformal symmetry.

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conformal symmetry + SUSY
→ superconformal symmetry.

Seiberg limit: in a SCFSC theory
γ^{α} always <0
→ also $-2\gamma^{\alpha} < 1$

→ $0 > \gamma^{\alpha} > -\frac{1}{2}$

$2\gamma^{\alpha} = 1 - \frac{3N_c}{N_f}$

$$\boxed{\text{→ conformal window of SQCD } \quad 3 > \frac{N_f}{N} > \frac{3}{2}}$$

SQCD phase diagram:

deep IR regimes of SQCD
with N_c colors & N_f <u>massless flavors</u>

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$\frac{N_f}{N_c} \geq 3$: non-abelian Coulomb phase.

$3 > \frac{N_f}{N_c} > \frac{3}{2}$: conformal window

$N_f = 0 \to$ pure SYM theory

$\to$ confinement, gluino bilinear condensation $\langle\lambda\lambda\rangle \neq 0$.

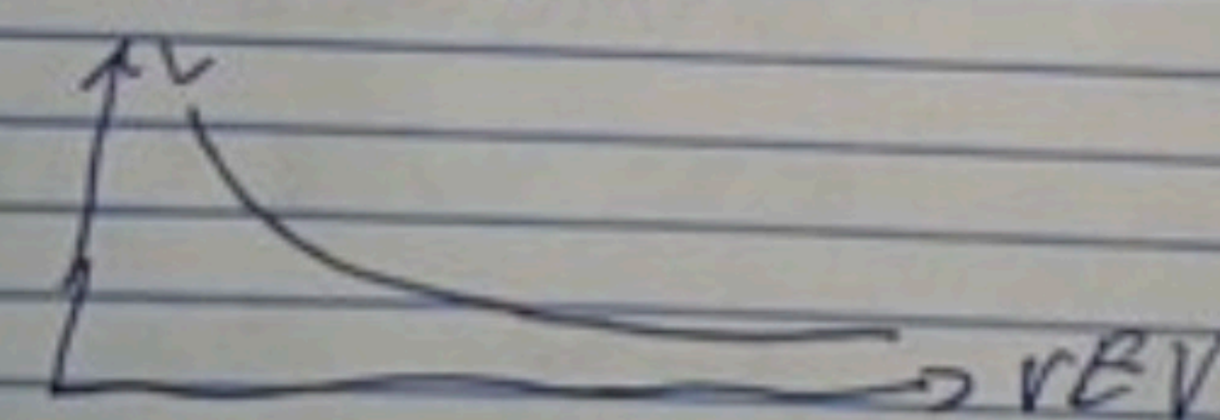
$N_f \geq 0$ but $N_f < N_c$ massless flavors,

no stable vacuum

squark VEV run away to ∞.

~~squark~~ non-pert scalar potential

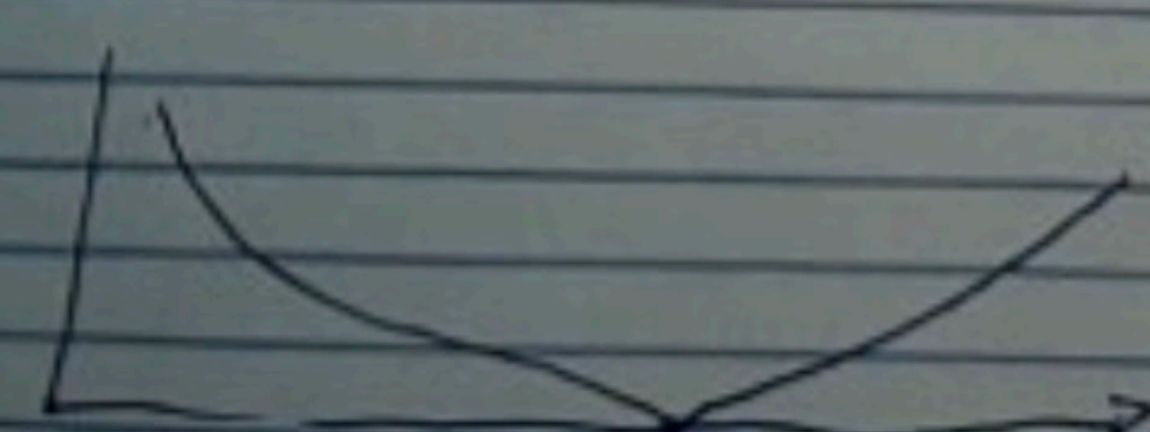
$V \sim \langle\phi\rangle^{\text{negative}}$



If the quarks are massive but light,

then there is a stable SUSY vacuum

where $\langle\phi\rangle \sim (m_Q)^{\text{negative}}$



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$N_f = N_c$ or $N_c + 1$ (massless flavors)

→ continuous family of SUSY vacua

$N_f < \frac{3}{2} N_c$ but $N_f > N_c + 1$

→ Seibers dual phase.

→ composite massless particles with SQCD' quantum numbers

$G = SU(N_c')$, $N_c' = N_f - N_c$

& $N_f' = N_f$ massless flavors

~~rest~~ gluons', gluinos', quarks', and squarks' are all composite.

$\frac{N_f'}{N_c'} > 3$ → SQCD' is in the non-abelian Coulomb phase.

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Beyond SQCD

Generalize ~~B~~NVSZ eqs to general gauge Theory

Gauge group $G = G_1 \otimes G_2 \otimes \cdots$

$\to$ gauge couplings $g_1, g_2, \cdots$

Chiral multiplets Φ_i in some <u>complete</u> multiplets $(r_1) + (r_2) + \cdots$ of G.

Assume: all triangle anomalies & trace anomalies cancel out. (for all gauge symmetries)

~~Also~~ Allow general but gauge-inv. superpotential.

beta-functions of gauge couplings

$$\beta^{1\text{loop}}[g_a] = B_a \times \frac{g_a^3}{16\pi^2}$$

regardless of all other couplings

$$B_a = -3R(\mathrm{adj}_{G_a}) + \sum_{(r_i)} R[G_a](r_i)$$

Wilsonian gauge couplings

$$\beta^{W}[g_a] = B_a \times \frac{g_a^3}{16\pi^2} \text{ exactly.}$$

8/8 Physical Running gauge couplings

NSVZ eq-s

$$\beta^{phys}[g_a] = \frac{g_a^3}{16\pi^2 - 2T(\mathrm{adj}_a)g_a^2} \times \left[-3T(\mathrm{adj}_a) + \sum_{(m)} T(G_a)(m) \times (1 - 2\gamma(m)) \right]$$

$T \equiv$ index

$T[G_a](m) =$ index (multiplet (m)) wrt gauge group G_a

In this f-la, the anomalous dimensions γ(mult. (m)) depend on all gauge & Yukawa ~~cou~~ couplings of the theory

---

Conditions for a fixed point of RG e. can be stated in terms of

$$\gamma_i(\text{couplings}) = \gamma_i^*$$

where γ_i^* are solutions to:

$\forall$ Yukawa $g_{ijk} \neq 0$, $\gamma_i^* + \gamma_j^* + \gamma_k^* = 0$

$\forall$ gauge couplings

$$-3T(\mathrm{adj}_a) + \sum_{(m)} T[G_a](m) \times (1 - 2\gamma^*_{(m)}) = 0.$$

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Example : $G = SU(N)$ gauge theory.

chiral matter : 3 adjoint multiplets

Φ^1, Φ^2, Φ^3 are $N \times N$ traceless matrices of chiral SF

$$W = y \, \mathrm{tr}(\Phi^1 [\Phi^2, \Phi^3])$$

$$= y \, \mathrm{tr}(\Phi^1\Phi^2\Phi^3 - \Phi^1\Phi^3\Phi^2)$$

↳ symmetric wrt Φ^1, Φ^2, Φ^3

$$\beta_y = y \times 3\gamma$$

same γ for all 3 Φ^1, Φ^2, Φ^3

$$\beta_g = \frac{g^3}{16\pi^2 - 2Ng^2}\left[-3N + 3N(1-2\gamma)\right]$$

$$[\,] = -6N\gamma \}$$

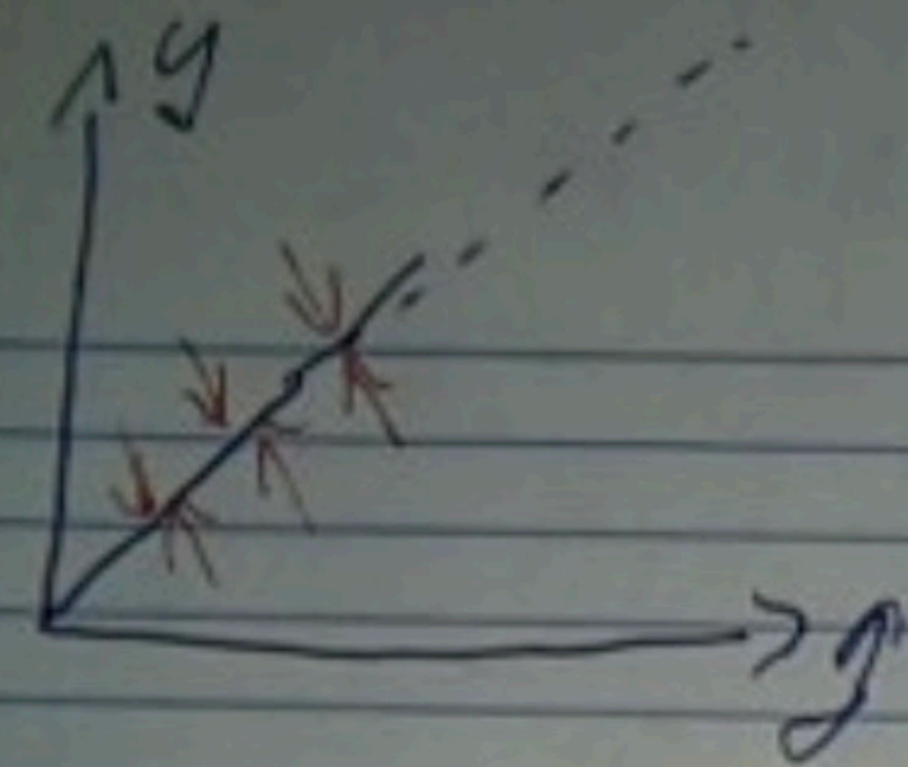
β_y = (positive) $\times \gamma$

β_g = (negative) $\times \gamma$

→ Fixed point when $\boxed{\gamma(g, y) = 0}$

$$\gamma = \frac{N}{8\pi^2}(|y|^2 - g^2) + \text{higher loops.}$$

2(c)



on the line, $\gamma = 0$

$\rightarrow \beta_g, \beta_y = 0$

below the line $\gamma < 0$

$\rightarrow \beta_y < 0, \beta_g > 0$

above the line, $\gamma > 0$

$\rightarrow \beta_y > 0, \beta_g < 0$

$\Rightarrow$ The $\gamma(y,g) = 0$ line is a line of IR-attractive fixed points:

$\forall$ starting point, in deep IR we end up somewhere on $\gamma = 0$ line

$\Rightarrow$ Superconformal Theory.

$\gamma = 0$ line starts @ $y = g$ $\rightarrow$ extended $\mathcal{N} = 4$ SUSY.

$\mathcal{N} = 4$ describes: only SYM,

in $\mathcal{N} = 1$ language: vectors in adj

+ 3 chiral SF Φ^1, Φ^2, Φ^3 in adj.

$\Rightarrow$ $y = g$ (modulo field redef).

8/11)

An $\mathcal{N}=1$ ~~Abog~~ theory with 3 adjoints but $y \neq g$

in a <u>deformed</u> $\mathcal{N}=4$ SYM.

RG flow to deep IR removes the deformation $\to$ deep IR has $\mathcal{N}=4$ superconformal symmetry

Any un-deformed $\mathcal{N}=4$ SYM is always superconformal, $\beta=0$, @ weak coupling or @ strong coupling $\Rightarrow$ @ any coupling g