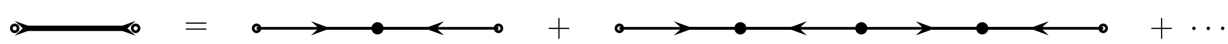
Problem **2**:

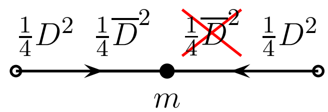
Consider the massive Φ to Φ propagator (5). Treating the mass as a perturbation of the massive theory, we get



(S.1)

Spelling out the first term on the RHS here, we get

1$^{\text{st}}$ term =



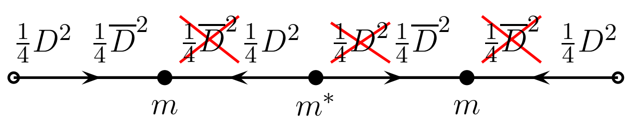
$$= \left(\frac{i}{p^2+i0}\right)^2 \times (-im) \times \tfrac{1}{64} D^2\overline{D}^2 D^2 \delta^4(\theta_1-\theta_2) \tag{S.2}$$
$$= \frac{im}{p^2+i0} \times \tfrac{1}{4} D^2 \delta^4(\theta_1-\theta_2),$$

where the last equality follows from

$$D^2\overline{D}^2 D^2 \;=\; 16p^2 \times D^2. \tag{S.3}$$

Similarly, spelling out the second term on the RHS of eq. (S.1), we get

2$^{\text{nd}}$ term =



$$= \left(\frac{i}{p^2+i0}\right)^4 \times (-im)^2(-im^*) \times \tfrac{1}{1024} D^2\overline{D}^2 D^2\overline{D}^2 D^2 \delta^4(\theta_1-\theta_2) \tag{S.4}$$
$$= \frac{im^2m^*}{(p^2+i0)^2} \times \tfrac{1}{4} D^2 \delta^4(\theta_1-\theta_2),$$

where the last equality follows from eq. (S.3) and hence

$$D^2\overline{D}^2 D^2\overline{D}^2 D^2 \;=\; (16p^2)^2 \times D^2. \tag{S.5}$$

In the same manner, the n^{th} term in the expansion of the massive propagator is the combination of $2n$ massless propagators and $2n-1$ mass vertices, which evaluates to

$$\begin{aligned} n^{\text{th}}\text{ term} &= \frac{im^n (m^*)^{n-1}}{(p^2+i0)^{2n}} \times \frac{1}{4^{2n+1}}\, D^2\overline{D}^2 \cdots D^2 \delta^4(\theta_1-\theta_2) \\ &= \frac{im^n (m^*)^{n-1}}{(p^2+i0)^{n}} \times \tfrac{1}{4} D^2 \delta^4(\theta_1-\theta_2). \end{aligned} \tag{S.6}$$

Consequently, summing over all the terms, we get the net massive $\Phi\Phi$ propagators to be

$$\begin{aligned} &= \sum_{n=1}^{\infty} \frac{im^n (m^*)^{n-1}}{(p^2+i0)^{n}} \times \tfrac{1}{4} D^2 \delta^4(\theta_1-\theta_2) \\ &= \left(\sum_n \left(\frac{mm^*}{p^2+i0} \right)^{n-1} \right) \times \frac{im}{p^2+i0} \times \tfrac{1}{4} D^2 \delta^4(\theta_1-\theta_2) \\ &= \frac{i}{p^2 - mm^* + i0} \times m \times \tfrac{1}{4} D^2 \delta^4(\theta_1-\theta_2), \end{aligned} \tag{S.7}$$

exactly as in eq. (5).

Finally, eq. (6) for the massive $\overline{\Phi\Phi}$ propagator obtains in exactly the same way. Simply repeat the above argument while reversing the directions of all the arrows and Hermitian conjugating all the formulae, thus $m \leftrightarrow m^*$ and $D^2 \leftrightarrow \overline{D}^2$.

<u>Problem **3**(b)</u>:

Consider a general superfield Feynman graph for the SQED. Suppose it has L loops, E_V external and P_V internal wavy lines belonging to the vectors, E_C external and P_C internal straight lines belonging to all types of chiral and antichiral superfields, and V_n vertices having n vector lines ($n = 1, 2, \ldots$) and 2 chiral lines; the net number of vertices is $V = \sum_n V_n$.

In the Feynman gauge, the superspace derivatives come only from the chiral propagators; there are 4 or 2 derivatives in each such propagator, depending on its type. On the other hand, the SQED vertices do not carry negative powers of D or $\overline{D}$, so the net number of superderivatives is

$$\#\text{SD} \;\leq\; 4P_C. \tag{S.8}$$

This inequality is saturated if all chiral propagators are of the $A\overline{A}$ or $\overline{B}B$ types; this is automatic when $m = 0$.

Since each SQED vertex has precisely two chiral lines, any graph has

$$2V \;=\; 2P_C \;+\; E_C\,. \tag{S.9}$$

Consequently, the number of the superderivatives is limited by

$$\#\mathrm{SD} \;\le\; 4V \;-\; 2E_C\,.$$

In a loop graph, $4L$ of these superderivatives are needed to close the loops, *i.e.* to eliminate the $\delta^{(4)}(\theta_1-\theta_2)\Big|_{\theta_1=\theta_2}$ factors in each loop. The remaining superderivatives may put loop momenta in the numerator (via the anticommutators $\{D_\alpha,\overline{D}_{\dot\alpha}\}=2q_{\alpha\dot\alpha}$) of the $\int d^{4L}q$ integral. In general, the numerator is a polynomial in loop momenta of degree

$$D_{\mathrm{num}} \;\le\; \tfrac{1}{2}\Big(\#\mathrm{SD} \;-\; 4L\Big) \;\le\; 2V \;-\; E_C \;-\; 2L \tag{S.10}$$

while the denominator has degree

$$D_{\mathrm{denom}} \;=\; 2P_{\mathrm{all}} \;=\; 2P_C \;+\; 2P_V \tag{S.11}$$

and the integral is over $4L$ momentum dimensions. This leads to the superficial degree of divergence

$$\mathcal{D} \;=\; 4L \;+\; D_{\mathrm{num}} \;-\; D_{\mathrm{denom}} \;\le\; 2L \;+\; 2V \;-\; 2P_{\mathrm{all}} \;-\; E_C\,. \tag{S.12}$$

By the Euler theorem $L+V-P_{\mathrm{all}}=1$, which gives us

$$\mathcal{D} \;\le 2 \;-\; E_C\,. \tag{7}$$

This result limits the divergent amplitudes of SQED to just two classes: (9) Two scalars (of opposite charges) and any number of vectors, and (8) just the vectors without any scalars at all. Both classes are infinite, which seems too much for a renormalizable theory. However, conservation of the supersymmetrized electric current leads to powerful Ward identities which drastically reduce the number of independent divergences from infinity to just two: $\delta Z_A=\delta Z_B$ and δe. I shall explain how this works later in class.