

Vector, Axial and Chiral Symmetries

Let $\Psi(x)$ be a Dirac spinor field $\Psi(x)$ with a free Lagrangian $\mathcal{L} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi$, and consider the bilinear currents

$$V^\mu \stackrel{\text{def}}{=} \bar{\Psi}\gamma^\mu\Psi \quad \text{and} \quad A^\mu \stackrel{\text{def}}{=} \bar{\Psi}\gamma^\mu\gamma^5\Psi. \quad (1)$$

As you should see in the [homework set#8](#), both V^μ and A^μ transform as 4-vectors under the continuous Lorentz symmetries, but under the space reflections the V^μ is a polar vector while A^μ is an axial vector. Consequently, the V^μ is called the *vector current* while the A^μ is called the *axial current*.

In a moment, we shall see that both the V^μ and the A^μ are Noether currents of the $U(1)_V \otimes U(1)_A$ symmetries of the Dirac field. These two symmetries are named after their respective currents: The $U(1)_V$ giving rise to the V^μ current is called the *vector symmetry* while the $U(1)_A$ giving rise to the A^μ current is called the *axial symmetry*.

Let's start with the vector symmetry $U(1)_V$, which is a global phase symmetry acting on the Dirac field and its conjugate as

$$\Psi(x) \rightarrow \Psi'(x) = e^{-i\theta}\Psi(x), \quad \bar{\Psi}(x) \rightarrow \bar{\Psi}'(x) = e^{+i\theta}\bar{\Psi}(x). \quad (2)$$

Its generator Q_V acts as $Q_V\Psi = -i\Psi$, $Q_V\bar{\Psi} = +i\bar{\Psi}$, so the Noether current is

$$\begin{aligned} J_{\text{Noether}}^\mu &= Q_V\bar{\Psi} \times \left. \frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\Psi})} \right|_{\text{left}}^{\text{from}} + \left. \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi)} \right|_{\text{right}}^{\text{from}} \times Q_V\Psi \\ &= i\bar{\Psi} \times 0 + i\bar{\Psi}\gamma^\mu \times (-i)\Psi \\ &= 0 + \bar{\Psi}\gamma^\mu\Psi, \end{aligned} \quad (3)$$

which is indeed the V^μ . And as any Noether current, the V^μ is conserved when the spinor fields obey the Dirac equations

$$\gamma^\mu \partial_\mu \Psi = -im\Psi, \quad \partial_\mu \bar{\Psi} \gamma^\mu = +im\bar{\Psi}. \quad (4)$$

Indeed,

$$\begin{aligned}
\partial_\mu(V^\mu = \bar{\Psi}\gamma^\mu\Psi) &= (\partial_\mu\bar{\Psi})\gamma^\mu \times \Psi + \bar{\Psi} \times \gamma^\mu\partial_\mu\Psi \\
\langle\langle \text{by eqs. (4)} \rangle\rangle &= im\bar{\Psi} \times \Psi + \bar{\Psi} \times (-im)\Psi \\
&= 0.
\end{aligned} \tag{5}$$

As to the axial $U(1)_A$ symmetry, it acts on the $\Psi(x)$ field as

$$\Psi(x) \rightarrow \Psi'(x) = \exp(-i\theta\gamma^5)\Psi(x), \tag{6}$$

and it's a symmetry of the Dirac Lagrangian only when the fermion mass m happens to vanish. In case you forgot your [homework set#7](#), the γ^5 in eq. (6) is the matrix $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$; it is hermitian, squares to 1, and anticommutes with all four γ^μ ,

$$\gamma^5\gamma^5 = 1, \quad \gamma^5\gamma^\mu = -\gamma^\mu\gamma^5. \tag{7}$$

Consequently

$$\exp(\pm i\theta\gamma^5)\gamma^\mu = \gamma^\mu \exp(\mp i\theta\gamma^5), \tag{8}$$

and hence

$$\begin{aligned}
\text{while } \Psi'^\dagger &= \Psi^\dagger \exp(+i\theta\gamma^5), \\
\text{but } \bar{\Psi}' &= \Psi'^\dagger\gamma^0 = \Psi^\dagger \exp(+i\theta\gamma^5)\gamma^0 = \Psi^\dagger\gamma^0 \exp(-i\theta\gamma^5) \\
&= \bar{\Psi} \exp(-i\theta\gamma^5).
\end{aligned} \tag{9}$$

Therefore, the kinetic term of the Dirac Lagrangian is invariant under the axial symmetry,

$$\begin{aligned}
\mathcal{L}_{\text{kin}} &= i\bar{\Psi}\gamma^\mu\partial_\mu\Psi \longrightarrow i\bar{\Psi}'\gamma^\mu\partial_\mu\Psi' = i\bar{\Psi} \exp(-i\theta\gamma^5)\partial_\mu \exp(-i\theta\gamma^5)\Psi \\
&= i\bar{\Psi} \times \exp(-i\theta\gamma^5)\gamma^\mu \exp(-i\theta\gamma^5) \times \partial_\mu\Psi \\
&= i\bar{\Psi}\gamma^\mu\partial_\mu\Psi = \mathcal{L}_{\text{kin}}
\end{aligned} \tag{10}$$

because

$$\exp(-i\theta\gamma^5)\gamma^\mu \exp(-i\theta\gamma^5) = \gamma^\mu \exp(+i\theta\gamma^5) \exp(-i\theta\gamma^5) = \gamma^\mu. \tag{11}$$

On the other hand, the fermion mass terms (for $m \neq 0$) is *not* invariant under the axial

symmetry:

$$\begin{aligned}
\mathcal{L}_{\text{mass}} &= -m\bar{\Psi}\Psi \longrightarrow -m\bar{\Psi}'\Psi' = -m\bar{\Psi}\exp(-i\theta\gamma^5) \times \exp(-i\theta\gamma^5)\Psi \\
&= -m\bar{\Psi}\exp(-2i\theta\gamma^5)\Psi \neq -m\bar{\Psi}\Psi = \mathcal{L}_{\text{mass}}
\end{aligned} \tag{12}$$

because $\exp(-2i\theta\gamma^5) \neq 1$. Thus, *only a massless Dirac fermion has the axial symmetry.*

Next, the Noether current of the axial symmetry. The generator Q_A acts on the spinor fields as $Q_A\Psi = -i\gamma^5\Psi$, $Q_A\bar{\Psi} = -i\bar{\Psi}\gamma^5$, hence the Noether current

$$\begin{aligned}
J_{\text{Noether}}^\mu &= Q_A\bar{\Psi} \times \frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\Psi})}\Big|_{\text{left}}^{\text{from}} + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\Psi)}\Big|_{\text{right}}^{\text{from}} \times Q_A\Psi \\
&= -i\bar{\Psi}\gamma^5 \times 0 + i\bar{\Psi}\gamma^\mu \times (-i\gamma^5)\Psi \\
&= 0 + \bar{\Psi}\gamma^\mu\gamma^5\Psi,
\end{aligned} \tag{13}$$

which is indeed the axial current A^μ . Consequently, this axial current should be conserved when the fields obey the Dirac equations (4), but only if the axial symmetry preserves the Lagrangian, *i.e.* only for the massless fermions. And indeed,

$$\begin{aligned}
\partial_\mu A^\mu &= \partial^\mu(\bar{\Psi}\gamma^\mu\gamma^5\Psi) = \partial_\mu\bar{\Psi}\gamma^\mu \times \gamma^5\Psi + \bar{\Psi}\gamma^\mu\gamma^5 \times \partial_\mu\Psi \\
&= \partial_\mu\bar{\Psi}\gamma^\mu \times \gamma^5\Psi - \bar{\Psi}\gamma^5 \times \gamma^\mu\partial_\mu\Psi \\
\langle\langle \text{by Dirac eqs.} \rangle\rangle &= +im\bar{\Psi} \times \gamma^5\Psi - \bar{\Psi}\gamma^5 \times (-im)\Psi \\
&= +2im\bar{\Psi}\gamma^5\Psi,
\end{aligned} \tag{14}$$

which vanishes for $m = 0$ but not for $m \neq 0$.

Now consider the $U(1)_V \otimes U(1)_A$ global symmetry from the Weyl spinor fields' point of view. In the Weyl convention for the γ^μ matrices

$$\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \quad \langle\langle \text{in } 2 \times 2 \text{ blocks} \rangle\rangle, \tag{15}$$

hence

$$\exp(-i\theta_V - i\theta_A\gamma^5) = \begin{pmatrix} e^{-i\theta_V+i\theta_A} & 0 \\ 0 & e^{-i\theta_V-i\theta_A} \end{pmatrix}. \tag{16}$$

In the same convention,

$$\Psi(x) = \begin{pmatrix} \psi_L(x) \\ \psi_R(x) \end{pmatrix} \quad (17)$$

where $\psi_L(x)$ and $\psi_R(x)$ are two-component Weyl spinor fields (left-handed ψ_L and right-handed ψ_R); in terms of these Weyl spinor fields, the *massless* Dirac Lagrangian is simply

$$\mathcal{L} = i\psi_L^\dagger \bar{\sigma}^\mu \partial_\mu \psi_L + i\psi_R^\dagger \sigma^\mu \partial_\mu \psi_R. \quad (18)$$

(I follow the Peskin&Schroeder convention where $\sigma^\mu = (1, +\boldsymbol{\sigma})$ and $\bar{\sigma}^\mu = (1, -\boldsymbol{\sigma})$.) In the Weyl spinor terms, the $U(1)_V \times U(1)_A$ symmetries act as

$$\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}' = \begin{pmatrix} e^{-i\theta_V + i\theta_A} & 0 \\ 0 & e^{-i\theta_V - i\theta_A} \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad (19)$$

meaning

$$\psi_L'(x) = e^{-i\theta_L} \psi_L(x), \quad \psi_R'(x) = e^{-i\theta_R} \psi_R(x) \quad (20)$$

for $\theta_L = \theta_V - \theta_A$ and $\theta_R = \theta_V + \theta_A$. In group theory terms, this means

$$U(1)_V \otimes U(1)_A = U(1)_L \otimes U(1)_R \quad (21)$$

where $U(1)_L$ acts only on the LH Weyl spinor while the $U(1)_R$ acts only on the RH Weyl spinor according to eq. (20). Such transforms — which are symmetries of the massless Lagrangian (18) for any independent θ_L and θ_R — are known as *chiral symmetries*, after the Greek word $\chi\epsilon\iota\rho$ for a hand. In Physics, chirality means a difference between a left-hand and a right-hand aspects of something; in our case, the different phases θ_L and θ_R for the LH and the RH Weyl spinor fields.

The Noether currents of the chiral $U(1)_L \otimes U(1)_R$ symmetry are the

$$J_L^\mu = \psi_L^\dagger \bar{\sigma}^\mu \psi_L \quad \text{and} \quad J_R^\mu = \psi_R^\dagger \sigma^\mu \psi_R^{\mathbf{P},s}, \quad (22)$$

or in terms of the Dirac spinor field Ψ ,

$$\begin{aligned} J_L^\mu &= \bar{\Psi} \gamma^\mu \frac{1 - \gamma^5}{2} \Psi = \frac{1}{2}(V^\mu - A^\mu), \\ J_R^\mu &= \bar{\Psi} \gamma^\mu \frac{1 + \gamma^5}{2} \Psi = \frac{1}{2}(V^\mu + A^\mu). \end{aligned} \quad (23)$$

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Now consider N Dirac fermions of the same electric charge q and same mass m ,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}_j(i\gamma^\mu\partial_\mu - q\gamma^\mu A_\mu - m)\Psi^j \quad \langle\langle \text{implicit } \sum_j \rangle\rangle. \quad (24)$$

This Lagrangian has an $U(N)_V$ group of global symmetries which act as

$$\Psi'^j(x) = U^j_k \Psi^k(x), \quad \bar{\Psi}'_j(x) = \bar{\Psi}_k(x)(U^\dagger)^k_j \quad (25)$$

for a unitary $N \times N$ matrix U . As a group $U(N)_V = U(1)_V \otimes SU(N)_V$, where the $U(1)_V$ is generated by the $Q : \Psi^j(x) \rightarrow -i\Psi^j(x)$ and the $SU(N)_V$ is generated by the $Q^a : \Psi_j(x) \rightarrow \frac{-i}{2}(\lambda^a)^j_k \Psi^k(x)$. Consequently, the Noether currents of the $U(1)_V \otimes SU(N)_V$ symmetries are

$$J^\mu = \bar{\Psi}_j \gamma^\mu \Psi^j \quad \text{and} \quad J^{\mu a} = \bar{\Psi}_j \gamma^\mu (\tfrac{1}{2}\lambda^a)^j_k \Psi^k. \quad (26)$$

Note: all these currents are true (polar) Lorentz vectors, that's why the $U(N)_V = U(1)_V \otimes SU(N)_V$ is indexed by the V . Together, these N^2 vector currents form an Hermitian matrix of currents

$$\begin{aligned} V^\mu &= \sum_a J^{\mu a} \times \lambda^a + \frac{1}{N} J^\mu \times 1_{N \times N}, \\ (V^\mu)^j_k &= \bar{\Psi}_k \gamma^\mu \Psi^j. \end{aligned} \quad (27)$$

For the massive fermions the only conserved currents are these vector currents, *but for the massless fermions we have a larger group of global symmetries and hence more conserved*

currents. Indeed, for each massless Dirac fermion Ψ^j , its left-handed and right-handed Weyl components ψ_L^j and ψ_R^j decouple from each other in the Lagrangian

$$\mathcal{L} = \psi_{L,j}^\dagger \bar{\sigma}^\mu (i\partial_\mu - qA_\mu) \psi_L^j + \psi_{R,j}^\dagger \sigma^\mu (i\partial_\mu - qA_\mu) \psi_R^j. \quad (28)$$

Consequently, we have separate $U(N)_L$ and $U(N)_R$ global symmetry groups where the $U(N)_L$ acts only on the LH Weyl fermions while the $U(N)_R$ acts only on the RH Weyl fermions:

$$\Psi_L'^j(x) = (U_L)^j_k \Psi_L^k(x), \quad \Psi_R'^j(x) = (U_R)^j_k \Psi_R^k(x), \quad (29)$$

for two independent $N \times N$ unitary matrices U_L and U_R . In terms of the Dirac fermions Ψ^j , the $U(N)_L \otimes U(N)_R$ *chiral symmetries* act as

$$\Psi'^j(x) = \left((U_L)^j_k \times \frac{1 - \gamma^5}{2} + (U_R)^j_k \times \frac{1 + \gamma^5}{2} \right) \Psi^k(x). \quad (30)$$

For $N > 1$, the finite chiral symmetries (30) cannot be reorganized as products of vector and axial symmetries, thus

$$U(N)_L \otimes U(N)_R \neq U(N)_V \otimes U(N)_A. \quad (31)$$

However, the generators of the chiral symmetries do combine into the vector and the axial generators, so the Noether currents also form linear combinations of the vector and the axial currents. To see how this works, consider the infinitesimal chiral transforms (30) for

$$U_L = 1 - i\epsilon_L + O(\epsilon_L^2), \quad U_R = 1 - i\epsilon_R + O(\epsilon_R^2) \quad (32)$$

where ϵ_L and ϵ_R are two independent infinitesimal Hermitian matrices. To the first order in

these infinitesimal matrices,

$$\begin{aligned}
\Psi'^j(x) &= \Psi^j(x) + \delta\Psi^j(x), \\
\delta\Psi^j(x) &= -i \left((\epsilon_L)^j_k \times \frac{1-\gamma^5}{2} + (\epsilon_R)^j_k \times \frac{1+\gamma^5}{2} \right) \Psi^k(x) \\
&= -i \left((\epsilon_V)^j_k + (\epsilon_A)^j_k \times \gamma^5 \right) \Psi^k(x)
\end{aligned} \tag{33}$$

for

$$\epsilon_V = \frac{1}{2}(\epsilon_R + \epsilon_L), \quad \epsilon_A = \frac{1}{2}(\epsilon_R - \epsilon_L). \tag{34}$$

As the infinitesimal parameters of chiral symmetries, we may replace the two independent Hermitian matrices ϵ_L and ϵ_R with two similarly independent hermitian matrices ϵ_V and ϵ_A . Consequently, we may immediately read the Noether currents from eq. (33): These currents form two hermitian $N \times N$ matrices

$$(V^\mu)^j_k = \bar{\Psi}_k \gamma^\mu \Psi^j \quad \text{and} \quad (A^\mu)^j_k = \bar{\Psi}_k \gamma^\mu \gamma^5 \Psi^j. \tag{35}$$

By inspection, the $(V^\mu)^j_k$ currents are true (polar) vectors while the $(A^\mu)^j_k$ currents are axial vectors.

The currents corresponding to the original $U(N)_L \otimes U(N)_R$ symmetries — and hence the ϵ_L and ϵ_R infinitesimal parameters rather than the ϵ_V and ϵ_A , — form linear combinations of the vector and the axial currents (35). Specifically, the $U(N)_L$ currents are the

$$(J_L)^j_k = \frac{1}{2}(V^\mu - A_\mu)^j_k = \psi_{L,k}^\dagger \bar{\sigma}^\mu \psi_L^j \tag{36}$$

while the $U(N)_R$ currents are the

$$(J_R)^j_k = \frac{1}{2}(V^\mu + A_\mu)^j_k = \psi_{R,k}^\dagger \sigma^\mu \psi_R^j. \tag{37}$$

Chiral Symmetries of QCD

Consider a QCD-like theory with N_c colors and N_f flavors — the non-abelian gauge theory with an $SU(N_c)$ gauge group and $N_F \times N_c$ Dirac fermions (quarks) $\Psi^{c,f}$ in N_F fundamental multiplets $\mathbf{N}_c$ of the gauge symmetry. The classical Lagrangian of this theory is

$$\mathcal{L} = -\frac{1}{4} \sum_{a=1}^{N_c^2-1} F_{\mu\nu}^a F^{a\mu\nu} + \sum_{f=1}^{N_f} \sum_{c=1}^{N_c} \bar{\Psi}_{c,f} (i\gamma^\mu D_\mu - m_f) \Psi^{c,f}, \quad (38)$$

where D_μ is the covariant derivative,

$$D_\mu \Psi^{c,f}(x) = \partial_\mu \Psi^{c,f} + \frac{ig}{2} \sum_a A_\mu^a(x) (\lambda^a)^c_{c'} \Psi^{c',f}(x). \quad (39)$$

Any global symmetry of the quark fields

$$\Psi'^{c,f}(x) = \sum_{c',f'} U^{c,f}_{c',f'} \times \Psi^{c',f'}(x) \quad (40)$$

must commute with all the covariant derivatives (39) for any gauge fields $A_\mu^a(x)$. Thus, the $N_c N_f \times N_c N_f$ matrix U must commute with all matrices λ^a acting only on the color indices, which means the U itself must act only on the flavor indices; furthermore, its action on the flavors must be independent on the quarks' colors. In other words, the global symmetries act as

$$\Psi'^{c,f}(x) = \sum_{f'} U_{f'}^f \times \Psi^{c,f'}(x) \quad \langle\langle \text{note same color } c \rangle\rangle \quad (41)$$

for a unitary $N_f \times N_f$ matrix U .

Moreover, the symmetries of the quarks must preserve their masses m_f . So if all quark flavors have different masses m_f , then the matrix $U_{f'}^f$ has to be diagonal, so the global symmetry group of the theory is simply $[U(1)_V]^{N_f}$. On the other hand, if all flavors have the same non-zero mass m , than any unitary matrix U acting on the flavors is a good symmetry, so the global symmetry group is $U(N_f)_V$. In both cases, the subscript V refers to the symmetry currents being true vectors $\bar{\Psi}\gamma^\mu\Psi$, because these are the only kinds of conserved currents that massive fermions might have.

But now suppose all the quark flavors are massless. In this case, all Dirac fermions $\Psi^{c,f}$ split into independent left-handed and right-handed Weyl fermions, so we have independent $U(N_f)_L$ and $U(N_f)_R$ symmetries acting on the flavors of the LH and the RH Weyl fermions. (But only on their flavors since all symmetries must preserve the covariant derivatives.) Altogether, the global symmetry group is the *chiral* $U(N_f)_L \otimes U(N_f)_R$ acting as

$$\Psi'^{c,f}(x) = \left((U_L)^f_{f'} \times \frac{1 - \gamma^5}{2} + (U_R)^f_{f'} \times \frac{1 + \gamma^5}{2} \right) \times \Psi^{c,f'}(x). \quad (42)$$

For $N_f > 2$ there are more patterns of quark masses, such as some flavors having the same mass but other flavors have different masses. Or some flavors being massless while other flavors being massive. But let me skip the symmetry analysis of such patterns: it should be obvious from the above, and you can figure it out when you need it.

Instead, let me focus on the real-life QCD (Quantum Chromodynamics, the theory of strong interactions) with $N_c = 3$ colors and $N_f = 6$ flavors. Formally, all six quark flavors have different masses, so the only exact flavor symmetry of the theory is the $[U(1)_V]^6$. But the effective strength of the QCD gauge coupling g depends on the energy scale of a process: moderate coupling at high energies but very strong coupling at low energies $E \lesssim \Lambda_{\text{QCD}} \sim 500 \text{ MeV}$. At low energies, there is no perturbative expansion in powers of g ; on the other hand, we may treat quark masses m_f which happens to be much less than Λ_{QCD} as small perturbations of the massless low-energy dynamics.

As it happens, two of the quark flavors — u and d — have very low masses, $m_u \approx 2.2 \text{ MeV}$ and $m_d \approx 4.7 \text{ MeV}$, much less than Λ_{QCD} . Consequently, to a good approximation we may neglect these two masses, which give the QCD a good *approximate chiral symmetry* $U(2)_L \times U(2)_R$. Moreover, the third flavor s is also fairly light, $m_s \approx 93 \text{ MeV}$, although not as light as u or d flavors. Nevertheless, $m_s \ll \Lambda_{\text{QCD}}$ so there is a useful approximation to the real-life QCD in which all three light flavors are massless, $m_u = m_d = m_s = 0$. In this approximation, the chiral symmetry becomes $U(3)_L \otimes U(3)_R$, although this approximate symmetry is not as good as the $U(2)_L \otimes U(2)_R$.

As to the remaining quark flavors c , b , and t , they are too heavy to neglect their masses, so they do not give rise to any approximate chiral symmetries.

To complete this section, let me tell you what happens to the various generators of the

$$U(2)_L \otimes U(2)_R = U(1)_V \otimes U(1)_A \otimes SU(2)_L \otimes SU(2)_R \quad (43)$$

chiral symmetry in the quantum rather than semiclassical chromodynamics:

- The $U(1)_V$ symmetry is exact, and its charge is the net quark number $= 3 \times$ the baryon number.
- The $U(1)_A$ symmetry is anomalous: it's explicitly broken by the quantum effects. I shall explain how such anomalies work in April.
- The $SU(2)_L \otimes SU(2)_R$ symmetry is *spontaneously broken* down to the $SU(2)_V$ isospin. In terms of its charges, the 3 vector charges T_V^a are realised in the Wigner mode (unbroken symmetries) while the 3 axial charges T_A^a are realized in the Goldstone mode (spontaneously broken). I shall explain the spontaneous breaking of symmetries in November; for the impatient, [here are my notes on the subject](#).

CHIRAL GAUGE THEORIES

Let's go back for a moment to N free Dirac fermions. If all N of these Dirac fermions are massless, then their LH and RH Weyl component become independent from each other, so the fermions have the chiral $U(N)_L \otimes U(N)_R$ symmetry. More generally, a massless theory may have different numbers $N_L \neq N_R$ of the left-handed and the right-handed Weyl spinor fields: without Dirac mass terms, the Weyl spinors do not have to pair up into Dirac spinors. In this case, the chiral symmetry becomes $U(N_L)_L \otimes U(N_R)_R$.

Now consider a gauge theory with some gauge group G . The couplings of the gauge fields to the fermions are completely determined by the assignments of the fermionic fields to the multiplets of the gauge group G . *In a chiral gauge theory, the LH Weyl fermions ψ_L^j belong to different multiplet types of G than the RH Weyl fermions ψ_R^j .*

The QCD is not chiral, since both ψ_L and ψ_R form fundamental triplets of $G = SU(3)_C$, and there are equal number of the LH and the RH triplets. On the other hand, *the electroweak $SU(2)_W \otimes U(1)_Y$ gauge theory is chiral*: the LH Weyl spinor fields of the Standard Model

form 12 $SU(2)_W$ doublets while the RH Weyl spinor fields form 24 singlets. Also the ψ_L and the ψ_R have different charges with respect to the $U(1)_Y$.

Consequently, any Grand Unified Theory (GUT) of strong and electroweak interactions must be chiral, because the Higgs mechanism cannot create chirality if it was not there to begin with. I shall return to this issue when I discuss the Grand Unification in an extra lecture sometimes in February. At the same time I shall give you a more careful definition of a chiral gauge theory, which takes into account the equivalence between the LH and the RH Weyl fields of opposite charges, $\psi_L^{\mathbf{P},s} \leftrightarrow \psi_R^\dagger$ and $\psi_R^{\mathbf{P},s} \leftrightarrow \psi_L^\dagger$.