This is an open-notes, open-book exam, so please do not waste your time copying parts of my class notes or homework solutions. If you need to use any homework result, simply reference the appropriate statement or equation and go ahead; likewise for the posted class notes, or anything else I have explained in class. Similarly, feel free to quote the textbook.

You may solve the exam problems in any order you like. If a problem looks too hard, do not hang up on it but go to the next problem, it might be easier. Then, once you have solved all the easy problems, go back to the hard problem(s).

1. Suppose the xy plane is conducting and carries a current sheet of uniform but time-dependent density $\mathbf{K} = K(t)\hat{\mathbf{x}}$. As you (should have) found in the homework set#8, the EM fields generated by this current sheet are

$$\begin{aligned} \mathbf{E}(z,t) &= -\frac{\mu_0 c}{2}\, K\big(t - \tfrac{|z|}{c}\big)\,\hat{\mathbf{x}}, \\ \mathbf{B}(z,t) &= -\frac{\mu_0\,\mathrm{sign}(z)}{2}\, K\big(t - \tfrac{|z|}{c}\big)\,\hat{\mathbf{y}}. \end{aligned} \tag{1}$$

Write down the energy density and the Poynting vector for these EM fields, then verify that they obey the Poynting theorem.

2. A circularly polarized plane EM wave hits a flat boundary between two transparent media with respective refraction indices n_1 and n_2. Describe the polarization of the reflected wave — is it circular, linear, or elliptic — in the following cases:

   (a) Normal (head-on) incidence, $\alpha_i = 0$.

   (b) Oblique incidence, *generic* $\alpha_i \neq 0$ for $n_1 < n_2$.

   (c) Incidence angle = Brewster angle.

   (d) Total internal reflection for $n_1 > n_2$ and large enough α_i.

3. A high-frequency harmonic current $I = I_0 \exp(-i\omega t)$ flows through a circular ring of radius a. Find the magnetic field $B(t)$ at the center of the ring.
Do not assume either $a\omega \ll c$ or $a\omega \gg c$.

4. A magnetic monopole of minimal magnetic charge $M = 2\pi\hbar/\mu_0 e$ is captured in a trap, where it oscillates with frequency ω and amplitude A. Find the EM power radiated by this oscillating monopole.

5. On January 1, 2050, a space probe is sent to a newly discovered planetary system X. The probe accelerates at constant rate $a = 9.5\ \text{m/s}^2 = c/(1\ \text{year})$ for 3 years (by the probe's clock), then decelerates at the same rate a for 3 more years (also by the probe's clock). As soon as the probe stops decelerating, it starts sending data about the system X back to Earth.

   (a) How far (in lightyears) from Earth is the system X?

   (b) When do the data from the probe start arriving to Earth?

6. In the rest frame of a conducting material, the current follows the electric field according to the Ohm's law, $\mathbf{J} = \sigma\mathbf{E}$, while the bulk charge density ρ is zero. Show that in the frame where the conductor moves at velocity $\mathbf{u}$

$$\mathbf{J} \;=\; \sigma\gamma\big(\mathbf{E} + \mathbf{u}\times\mathbf{B}\big). \tag{2}$$