PHY–352 L. Problem set #7 Due March 5, 2026 (Thursday).

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1. A particle of charge Q moves in a circle of radius a at constant angular velocity ω. Use the coordinate system where that circle lies in the xy plane and its center is at the coordinate origin. Find the Liénard–Wiechert potentials V and $\mathbf{A}$ for points on the z axis of that coordinate system.

2. [15 points] A particle moves in a hyperbolic fashion along the x axis,

$$w_x(t) = \sqrt{b^2 + (ct)^2}\,, \quad w_y(t) = w_z(t) = 0 \quad \text{for all } -\infty < t < +\infty. \tag{1}$$

(This is the relativistic version of motion at a constant acceleration $a = c^2/b$.)
Consider an observer located on the x axis, to the left of point b.

   (a) Show that for $t \leq -x/c$, the observer cannot see the particle.

   (b) For $t > -x/c$, the observer does see the particle. Find the retarded time at which the particle is seen and the particle's location at that time.

   (c) Calculate the Liénard–Wiechert potentials at the observer point.

3. A point charge Q moves in a straight line along the x axis, but its velocity $v(t)$ is time-dependent.

   (a) Show that the EM fields at points lying on the x axis to the right of the charge are

$$\mathbf{E}(x,t) = \frac{Q\hat{\mathbf{x}}}{4\pi\epsilon_0}\left[\frac{1}{\mathcal{R}^2}\left(\frac{c+v}{c-v}\right)\right]_{\text{ret}} \qquad \mathbf{B}(x,t) = \mathbf{0}. \tag{2}$$

   (b) Find the EM fields at points lying on the same x axis but to the left of the charge.

4. A particle of charge Q travels at constant velocity v along the z axis. Find the net EM power flowing through the xy plane at the time the particle itself is located at $\mathbf{w} = -a\hat{\mathbf{z}}$.

5. [20 points] When 2 particles move relative to each other, the forces between them do not obey Newton's Third Law $\mathbf{F}_{12} = -\mathbf{F}_{21}$ because these forces include momentum exchange with the EM fields surrounding the particles. Thus, in order to conserve the *net* momentum of both EM fields and the two particles, we should have

$$\mathbf{F}_{12}(t) \;+\; \mathbf{F}_{21}(t) \;+\; \frac{d\mathbf{p}_{\text{EM}}}{dt} \;=\; 0. \tag{3}$$

To check this rule, consider the following system: One point charge Q_1 is at rest at the coordinate origin, while another point charge Q_2 moves at constant velocity along the z axis, $\mathbf{r}_2(t) = vt\hat{\mathbf{z}}$.

(a) Find the force $\mathbf{F}_{12}(t)$ by Q_1 on Q_2 at time t.

(b) Find the force $\mathbf{F}_{21}(t)$ by Q_2 on Q_1 at the same time t.

(c) Show that the net EM momentum at time t is

$$\mathbf{p}_{\text{EM}}(t) \;=\; \frac{Q_1 Q_2 \mu_0}{4\pi t}\,\hat{\mathbf{z}} \;+\; \text{constant}. \tag{4}$$

Note: the constant here is divergent, as its originates from the divergent electrostatic energy of the point charge Q_2 which moves along with it at the velocity $v\hat{\mathbf{z}}$. Fortunately, for our purposes we don't need this time-independent constant.
Mathematical hint:

$$\int_0^\infty \frac{s^3\,ds}{(s^2+a^2)^{3/2}(s^2+b^2)^{3/2}} \;=\; \frac{1}{(|a|+|b|)^2}\,. \tag{5}$$

(d) Check that the two forces and the net EM momentum obey eq. (3).

6. [5 points] A proton is released from rest and is falling down due to gravitational force. While it falls through the first centimeter, what fraction of its potential energy is lost to the EM radiation?

7. [15 points] A positive charge q is fired head-on at a distant positive charge Q (which is held stationary) with initial velocity v_0. It comes in, decelerates to $v = 0$, and returns to the infinity. What fraction of its initial kinetic energy $\frac{1}{2}mv_0^2$ is radiated away? Assume $v_0 \ll c$, and ignore the effect of the radiative losses on the particle's motion.

   Hint: don't try to work out the time-dependence of the particle's motion, it's too messy. Instead, find the particle's acceleration as a function of its velocity, then use $a^2\,dt = a\,dv$.

8. Consider an electron in a harmonic potential $U(x) = \frac{1}{2}m\omega_0^2 \times x^2$. Due to Abraham–Lorentz radiation reaction force on the electron, this is a dumped oscillator. Find the quality factor of this dumped oscillator.
   See homework set#2, problem 3 for the definition of the quality factor.