1. Consider a radio link between two half-dipole antennas $R = 20$ km apart from each other. Each antenna is $h = 2$ m high vertical wire, with the image charge of that wire in the ground acting as the second half of the corresponding dipole. The first antenna transmits a 5 MHz signal ($\lambda = 60$ m) at the net power of $P = 1$ kW. How much of that power is received by the second antenna and delivered to the tuner? Assume the tuner's input impedance is matched to the antenna's.

2. [15 points] In Galilean relativity, the time is absolute but the velocities are relative to a frame of reference. When comparing velocities $\mathbf{v}_{o1}$ and $\mathbf{v}_{o2}$ of the same object $\mathcal{O}$ relative to two different frames K_1 and K_2, the velocities add as vectors,

$$\mathbf{v}_{o2} = \mathbf{v}_{o1} + \mathbf{u}_{12} \tag{1}$$

where $\mathbf{u}_{12}$ is the velocity of the frame K_1 relative to the frame K_2.
Of particular importance are the *inertial frames of reference* in which a particle free from any forces stays at rest or moves at constant velocity vector $\mathbf{v}$; the First Law of Newton postulates such frames do exist.

   (a) Show that any two inertial frames K_1 and K_2 must move at a constant velocity vector relative to each other. Also show that if the frame K_1 is inertial and the frame K_2 moves at a constant velocity relative to the K_1, then the K_2 frame is also inertial.

   Now consider a two-particle collision in two inertial frames of reference, K_1 and K_2. In the frame K_1, particle A has initial velocity $\mathbf{v}_A$ and mass m_A while the particle B has initial velocity $\mathbf{v}_B$ and mass m_B. During the collision, the two particles exchange not only some of their momenta but also sum of their masses: a bit of particle A's mass is broken off and sticks to particle B. So after the collision particle A has a different velocity $\mathbf{v}'_A$ and a different mass m'_A, and likewise particle B has velocity $\mathbf{v}'_B$ and mass m'_B. Nevertheless, the net mass of the two particles is conserved, and the net momentum is also conserved,

$$m'_A + m'_B = m_A + m_B\,, \tag{2}$$

$$m'_A \mathbf{v}'_A + m'_B \mathbf{v}'_B = m_A \mathbf{v}_A + m_B \mathbf{v}_B . \tag{3}$$

(b) Show that if the net momentum is conserved in some inertial frame K_1 then it is also conserved in any other inertial frame K_2.

(c) Show if the collision is *elastic* — *i.e.*, the net kinetic energy is conserved — in one inertial frame then it's also elastic in any other inertial frame.

3. [15 points] In Einstein's special relativity, the time is relative while the speed of light (in the vacuum) is absolute, so the Galilean rule (1) for adding velocities becomes

$$v_{o2} = \frac{v_{o1} + u_{12}}{1 + (v_{o1} u_{12})/c^2} \tag{4}$$

(when all 3 velocities are along the same axis).

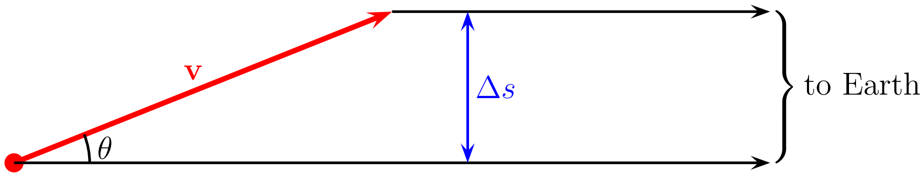
(a) An airplane flies at airspeed 250 m/s (560 MPH) in a 50 m/s (112 MPH) tail wind, and you want to find its ground-speed. How much error would you make using the Galilean formula (1) instead of the Einstein's formula (4)?

(b) Show that for any two frames moving relative to each other with a velocity less than c, if a body has velocity less then c relative to one frame then it's velocity relative to the other frame is also less then c.
What is the physical meaning of this result?

(c) Now imagine a world where the speed of light is much slower than in ours, say $c = 300$ km/h instead of $300,000$ km/s. In that world, you are standing on the floor of a train moving at speed $\frac{1}{2}c$ and shoot a gun in the forward direction. The bullet's velocity relative to the gun is $\frac{2}{3}c$. What is its velocity relative to the ground?

4. Every now and then, an amateur astronomer discovers an object which seems to move across the sky at a faster than light velocity. While some of these 'discoveries' are simply wrong, other are genuine observations but fail to distinguish between the observer time in which they see the motion and the retarded time in which the object moves and emits light they see. For an object moving at high speed (but slower than light) towards the observer on

Earth, the observer times runs much slower than the retarded time, so the object seems to move much faster than it actually does.



(5)

(a) Calculate the apparent $\perp$ speed of the object $\Delta s/\Delta t_{\text{obs}}$ in terms of its actual speed v and direction θ.

Note: this problem has nothing to do with the relativistic Lorentz contraction and time dilation. All you need here is the geometry and the relation between the observer time and the retarded time.

(b) For a given actual velocity $v < c$ of the object, what angle θ yields the highest apparent velocity?

(c) Show that the apparent velocity can be faster than light — and even much faster than light — despite actual velocity being slower than light.

5. A π^+ meson has an average lifetime of 26 ns. In a beam of such pions produced at an accelerator lab, the pions travel an average distance of 13 m before decaying. A naive estimate of the pions' velocity based on these data yields

$$v = \frac{\Delta L_{\text{avg}}}{\Delta t_{\text{avg}}} = \frac{13\text{ m}}{26\text{ ns}} = 5\cdot 10^8\text{ m/s}, \tag{6}$$

faster then light!

(a) What is the mistake leading to this estimate?

(b) Find the actual speed of the pions.

6. A starship travels at constant velocity $v = \frac{4}{5}c$. At the time the ship was launched from Earth, both shipboard clock and Earthbound clock were set to $t = 0$. Then, when the shipboard clock showed $t = 1$ hour, a radio signal was sent back to Earth.

(a) When was the signal sent according to the Earth clock?

(b) When did the signal reach Earth by the Earth clock?

(c) When did the signal reach Earth by the shipboard clock?

7. Two starships fly through the solar system without stopping, and to the Earthbound observers they appear to have similar lengths. The observers have identified the two ship's makes, and according to the Galactic Catalogue ship A should be twice as long as ship B (when they are measured in their own frames). The speed of ship B (relative to the Earth) was measured to be $\frac{1}{2}c$. What was the speed of ship A?