This is an open-notes, open-book exam, so please do not waste your time copying parts of my class notes or homework solutions. If you need to use any homework result, simply reference the appropriate statement or equation and go ahead; likewise for the posted class notes, or anything else I have explained in class. Similarly, feel free to quote the textbook.

1. Consider the potentials

$$\begin{aligned} V(x,y,z;t) &= Dtz, \\ \mathbf{A}(x,y,z;t) &= \frac{D}{4c^2}\,(x^2+y^2)\,\hat{\mathbf{z}}, \end{aligned} \tag{1}$$

for some constant D.

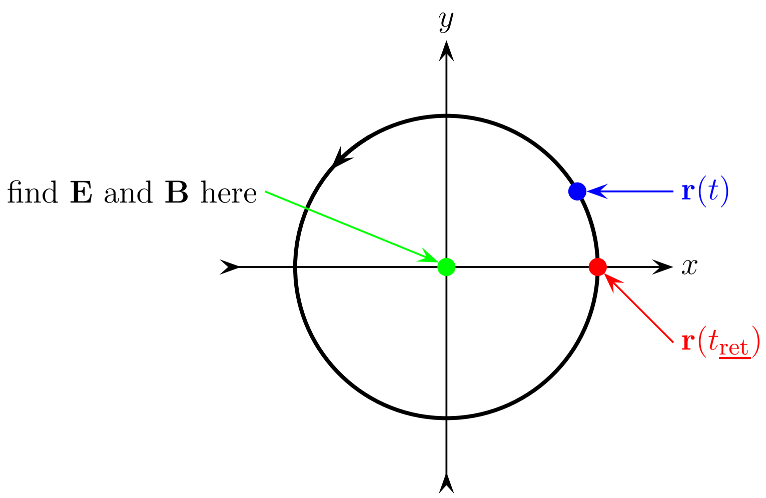
(a) Do these potentials obey the Coulomb gauge or the Landau gauge conditions? Or both? Or neither?

(b) Write down the electric and the magnetic fields $\mathbf{E}$ and $\mathbf{B}$ for these potentials.

(c) Do these EM fields obey the Maxwell equations for the vacuum with $\rho = 0$ and $\mathbf{J} = 0$?

2. A point charge Q moves in a circle or radius b at a constant speed v. Assume $v < c$ but do not assume $v \ll c$. Find the electric and the magnetic fields at the center of the circle.

For simplicity, you may choose a particular time t and calculate the EM fields at the center only for that time t. I recommend choosing t at which



(2)

3. Consider an inductor coil with a ferrite core used as a magnetic-dipole antenna. Such 'ferrite loopstick' antennas are often used in compact radio receivers, but they may be also used for low-power transmissions.

   For the sake of definiteness, let the ferrite core have length $\ell = 10$ cm, cross-section $A = 1.00$ cm^2, and $\mu = 1500$, while the coil around the core has $N = 60$ turns. The antenna broadcasts on a citizen-band frequency $f = 27$ MHz ($\lambda \approx 11$ m) at power $P = 4$ W. Find the amplitude of the current powering this broadcast.

4. An electron moves in a circle in a uniform magnetic field B. As the electron radiates EM waves, it slows down and the radius R of its orbit becomes smaller. Show that (for a non-relativistic electron speed) R decreases as

$$R(t) \;=\; R_0 \times \exp(-t/\tau) \tag{3}$$

   and calculate the time constant τ as a function of the magnetic field B and universal constants.

   ⋆ For a small extra credit, calculate the numeric value of the τ for $B = 1.00$ Tesla. FYI, electron's mass is $m_e = 0.911 \cdot 10^{-30}$ kg, $e = 1.60 \cdot 10^{-19}$ C, $\mu_0 = 4\pi \cdot 10^{-7}$ T/(A/m), and $c = 3.00 \cdot 10^8$ m/s.