Problem **1**:

In the reference frame K of the Earth, the two events — J (John stumbling) and M (Mary crying) — happen at respective spacetime points $x_J = L,\ ct_J = 0$ and $x_M = 0,\ ct_M = 0$ where $L = 384,000$ km is the distance from the Earth to the Moon. In the same Earth frame, the alien probe moves in the *negative* x direction, so in the reference frame K' of the probe, the Earth moves in the *positive* x direction. Consequently, Lorentz-transforming to the K' frame, we find that Mary cries at

$$\begin{aligned} x'_M &= \gamma x_M + \beta\gamma c t_M = 0, \\ ct'_M &= \gamma c t_M + \beta\gamma x_M = 0, \end{aligned} \tag{S.1}$$

while John stumbles at

$$\begin{aligned} x'_J &= \gamma x_J + \beta\gamma c t_J = \gamma L + 0 = \gamma L, \\ ct'_J &= \gamma c t_J + \beta\gamma x_J = 0 + \beta\gamma J = +\beta\gamma L. \end{aligned} \tag{S.2}$$

Thus, we see that in the frame of the probe, $t_J > t_M$, so Mary cries *before* John stumbles.

As to the time difference between the two events in the probe frame,

$$\Delta t = t_J - t_M = \frac{\beta\gamma L}{c} - \frac{0}{c} = \beta\gamma \times \frac{L}{c}\,, \tag{S.3}$$

where

$$\frac{L}{c} = \frac{384,000\ \text{km}}{300,000\ \text{km/s}} = 1.28\ \text{s}, \tag{S.4}$$

$$\beta = \frac{v_{\text{probe}}}{c} = \frac{12}{13}\,, \tag{S.5}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{13}{\sqrt{13^2 - 12^2}} = \frac{13}{5}\,, \tag{S.6}$$

hence

$$\Delta t = 1.28\ \text{s} \times \frac{12}{13} \times \frac{13}{5} \approx 3.07\ \text{s}. \tag{S.7}$$

<u>Problem</u> **2**:

(a) Reversing eq. (2) we have

$$\beta \;=\; \frac{v}{c} \;=\; \tanh(w), \tag{S.8}$$

hence

$$\gamma \;=\; \frac{1}{\sqrt{1-\tanh^2(w)}} \;=\; \frac{\cosh(w)}{\sqrt{\underline{\underline{\cosh^2(w)-\sinh^2(w)=1}}}} \;=\; \cosh(w) \tag{S.9}$$

and therefore

$$\beta\gamma \;=\; \tanh(w)\times\cosh(w) \;=\; \sinh(w). \tag{S.10}$$

Consequently, the Lorentz transform matrix for a boost in x direction becomes

$$L^\mu{}_\nu \;=\; \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \;=\; \begin{pmatrix} \cosh(w) & \sinh(w) & 0 & 0 \\ \sinh(w) & \cosh(w) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{S.11}$$

By comparison, the rotation matrix for a rotation through angle ϕ around the z axis is

$$R^{ij} \;=\; \begin{pmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ +\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{S.12}$$

We see that the Boost matrix (S.11) is quite similar to the rotation matrix (S.12), but there a couple of key differences (apart from the extra row and extra column of the Lorentz boost matrix). Specifically, (1) the boost matrix involves hyperbolic rather than trigonometric sine and cosine; and (2) in the boost matrix both non-zero off-diagonal matrix elements come with the same sign '+' unlike the rotation matrix where the two off-diagonal elements have opposite signs. Both of these differences are caused by the pseudo-Euclidean rather than Euclidean geometry of the spacetime.

(b) The hyperbolic sine, cosine, and tangent of a sum obtain as

$$\begin{aligned}
\cosh(w_1+w_2) &= \cosh(w_1)\cosh(w_2) \;+\; \sinh(w_1)\sinh(w_2), && \text{(S.13)}\\
\sinh(w_1+w_2) &= \sinh(w_1)\cosh(w_2) \;+\; \cosh(w_1)\sinh(w_2), && \text{(S.14)}\\
& \text{note the positive signs in both formulae,} \\
\tanh(w_1+w_2) &= \frac{\sinh(w_1+w_2)}{\cosh(w_1+w_2)} = \frac{\cosh(w_1)\cosh(w_2)\times\bigl(\tanh(w_1)+\tanh(w_2)\bigr)}{\cosh(w_1)\cosh(w_2)\times\bigl(1\,+\,\tanh(w_1)\tanh(w_2)\bigr)}\\
&= \frac{\tanh(w_1)+\tanh(w_2)}{1\,+\,\tanh(w_1)\tanh(w_2)}\,. && \text{(S.15)}
\end{aligned}$$

At the same time, combining two Lorentz boosts of velocities v_1 and v_2 in the same direction, we get a boost of net velocity

$$v_{1+2} = \frac{v_1+v_2}{1\,+\,\frac{v_1v_2}{c^2}}\,. \tag{S.16}$$

In terms of the respective rapidities w_1 and w_2,

$$v_1 = c\tanh(w_1), \qquad v_2 = c\tanh(w_2), \tag{S.17}$$

hence

$$v_{1+2} = \frac{c\tanh(w_1)\,+\,c\tanh(w_2)}{1\,+\,\tanh(w_1)\tanh(w_2)} = c\tanh(w_1+w_2) \tag{S.18}$$

where the second equality follows from eq. (S.15) for the hyperbolic tangents. According to eq. (S.18), for successive Lorentz boosts in the same directions, the rapidities indeed add up,

$$w_{1+2} = w_1 \,+\, w_2\,. \tag{S.19}$$

Problem **3**:

Let me start with a more general case: a Lorentz boost by some velocity $\mathbf{v} = c\boldsymbol{\beta}$ in a general direction. For such a boost, the Lorentz transform of the coordinates is

$$\begin{aligned} x'^0 &= \gamma x^0 \;+\; \gamma\boldsymbol{\beta}\cdot\mathbf{x}, \\ \mathbf{x}' &= \gamma\boldsymbol{\beta}x^0 \;+\; \mathbf{x} \;+\; \frac{\gamma-1}{\beta^2}(\boldsymbol{\beta}\cdot\mathbf{x})\boldsymbol{\beta}. \end{aligned} \tag{S.20}$$

In terms of the Lorentz matrix $L^\mu_{\ \nu}$ such that $x'^\mu = L^\mu_{\ \nu}x^\nu$, eq. (S.20) means

$$\begin{aligned} L^0_{\ 0} &= \gamma, \\ L^0_{\ j} &= \gamma\beta^j, \\ L^i_{\ 0} &= \gamma\beta^i, \\ L^i_{\ j} &= \delta^{ij} \;+\; \frac{\gamma-1}{\beta^2}\,\beta^i\beta^j, \end{aligned} \tag{S.21}$$

hence in matrix form

$$L^\mu_{\ \nu} \;=\; \begin{pmatrix} \gamma & \gamma\beta^x & \gamma\beta^y & \gamma\beta^z \\ \gamma\beta^x & 1+F(\beta^x)^2 & F\beta^x\beta^y & F\beta^x\beta^z \\ \gamma\beta^y & F\beta^y\beta^x & 1+F(\beta^y)^2 & F\beta^y\beta^z \\ \gamma\beta^z & F\beta^z\beta^x & F\beta^z\beta^y & 1+F(\beta^z)^2 \end{pmatrix} \tag{S.22}$$

where

$$F \;=\; \frac{\gamma-1}{\beta^2} \;=\; \frac{\gamma^2}{\gamma+1}\,. \tag{S.23}$$

Now let's apply eq. (S.22) to the velocity vector (4). For this vector

$$\beta^x \;=\; \beta\cos(\phi), \qquad \beta^y \;=\; \beta\sin(\phi), \qquad \beta^z \;=\; 0, \tag{S.24}$$

hence

$$\begin{aligned} 1 \;+\; F(\beta^x)^2 &= 1 \;+\; (\gamma-1)\left(\frac{(\beta^x)^2}{\beta^2} \;=\; \cos^2\phi\right), \\ 1 \;+\; F(\beta^y)^2 &= 1 \;+\; (\gamma-1)\sin^2\phi, \\ F\beta^x\beta^y &= (\gamma-1)\cos\phi\sin\phi, \end{aligned} \tag{S.25}$$

and therefore

$$L^{\mu}_{\ \nu} \;=\; \begin{pmatrix} \gamma & \gamma\beta\cos\phi & \gamma\beta\sin\phi & 0 \\ \gamma\beta\cos\phi & 1+(\gamma-1)\cos^2\phi & (\gamma-1)\cos\phi\sin\phi & 0 \\ \gamma\beta\sin\phi & (\gamma-1)\cos\phi\sin\phi & 1+(\gamma-1)\sin^2\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{S.26}$$

Problem **4**:

(a)

$$\begin{aligned} I^2 &= (ct_A - ct_B)^2 \;-\; (x_A - x_B)^2 \;-\; (y_A - y_B)^2 \;-\; (z_A - z_B)^2 \\ &= (15-5)^2 \;-\; (5-10)^2 \;-\; (3-8)^2 \;-\; (0-0)^2 \\ &= 100 \;-\; 25 \;-\; 25 \;-\; 0 \;=\; +50 \end{aligned} \tag{S.27}$$

hence real $I = +\sqrt{50}$. Note that this is a timelike interval.

(b) Since the interval between the two events is timelike, there is no inertial frame in which they happen at the same time. Instead, in all frames the event A happens after the event B by at least $c\Delta t = I = \sqrt{50}$¿

(c) Since the interval between the two events is timelike, there exists an inertial frame K'' where both events occur at the same place (but different times). The velocity of that frame relative to the K frame is simply

$$\mathbf{v}'' \;=\; \frac{\mathbf{r}_A - \mathbf{r}_B}{t_A - t_B} \;=\; c\frac{(5,3,0)-(10,8,0)}{15-5} \;=\; c(-\tfrac{1}{2},-\tfrac{1}{2},0). \tag{S.28}$$

Indeed, for

$$\boldsymbol{\beta} \;=\; \frac{\Delta\mathbf{r}}{\Delta t}\,, \tag{S.29}$$

the Lorentz transformed $\Delta\mathbf{r}'$ becomes

$$\Delta\mathbf{r}' \;=\; -\gamma\boldsymbol{\beta}\Delta t \;+\; \Delta\mathbf{r} \;+\; (\gamma-1)\frac{(\boldsymbol{\beta}\cdot\Delta\mathbf{r})\boldsymbol{\beta}}{\beta^2} \;=\; -\gamma\Delta\mathbf{r} \;+\; \Delta\mathbf{r} \;+\; (\gamma-1)\Delta\mathbf{r} \;=\; 0. \tag{S.30}$$

(d) This time

$$\begin{aligned} I^2 &= (ct_C - ct_D)^2 \;-\; (x_C - x_D)^2 \;-\; (y_C - y_D)^2 \;-\; (z_C - z_D)^2 \\ &= (1-3)^2 \;-\; (2-5)^2 \;-\; (0-0)^2 \;-\; (0-0)^2 \qquad \text{(S.31)} \\ &= 4 \;-\; 9 \;-\; 0 \;-\; 0 \;=\; -5, \end{aligned}$$

so the interval is timelike, $I = \sqrt{5}i$.

For such a timelike interval, there is no inertial frame K'' where both events happen at the same place (but at different times). On the other hand, there are frames where both events happen at the same time (but at different places). To find such a frame, we solve for

$$c\Delta t' \;=\; \gamma c\Delta t \;-\; \gamma\boldsymbol{\beta}\cdot\Delta\mathbf{r} \;=\; 0, \tag{S.32}$$

hence

$$\boldsymbol{\beta}\cdot\Delta\mathbf{r} \;=\; c\Delta t. \tag{S.33}$$

For the events at hand, $\Delta\mathbf{r} = (-3,0,0)$, $c\Delta t = -2$, so we need

$$\beta^x \;=\; \frac{2}{3} \tag{S.34}$$

while β^y and β^z can be anything as long as $\boldsymbol{\beta}^2 < 1$. In particular, the simplest solution is $\mathbf{v}' = \frac{2}{3}c\hat{\mathbf{x}}$.

Problem **5**:

(a) In light of eq. (5)

$$u^0 \;=\; \gamma c, \qquad \mathbf{u} \;=\; \gamma\mathbf{v}, \tag{S.35}$$

hence

$$\mathbf{v} \;=\; \frac{c}{u^0}\,\mathbf{u}\,. \tag{S.36}$$

Although we are not given the u^0 component of the proper velocity but only the space components $\mathbf{u}$, we may reconstruct the u^0 from the constraints

$$(u \cdot u) \;=\; (u^0)^2 \;-\; \mathbf{u}^2 \;=\; c^2 \quad \text{and} \quad u^0 \;>\; 0. \tag{S.37}$$

Solving these constraints we immediately get

$$u^0 \;=\; +\sqrt{c^2 + \mathbf{u}^2} \tag{S.38}$$

and therefore

$$\mathbf{v} \;=\; \frac{c}{\sqrt{c^2 + \mathbf{u}^2}}\,\mathbf{u}. \tag{S.39}$$

(b) Since the particle moves in the $\hat{\mathbf{x}}$ direction, $v^y = v^z = 0$, while v^x obtains from the rapidity w as

$$v^x \;=\; c\tanh(w). \tag{S.40}$$

Consequently,

$$\begin{aligned} |\beta| \;&=\; \frac{|v|}{c} \;=\; \tanh(w), \\ \gamma \;&=\; \frac{1}{\sqrt{1 - \tanh^2(w)}} \;=\; \cosh(w), \\ \gamma\beta =&\; \sinh(w), \end{aligned} \tag{S.41}$$

and therefore

$$\begin{aligned} u^0 \;&=\; c\gamma \;=\; c\cosh(w), \\ u^x \;&=\; c\beta\gamma \;=\; c\sinh(w), \\ u^y \;&=\; u^z \;=\; 0. \end{aligned} \tag{S.42}$$

Altogether,

$$u^\mu \;=\; \big(c\cosh(w); c\sinh(w), 0, 0\big). \tag{S.43}$$

Problem **6**:

(a) Since $\cos(45°) = \sin(45°) = \sqrt{\frac{1}{2}}$, the particle's 3–velocity vector in the K frame is

$$\mathbf{v} \;=\; \left(\sqrt{\tfrac{1}{2}}v, \sqrt{\tfrac{1}{2}}v, 0\right) \;=\; \left(\sqrt{\tfrac{2}{5}}c, \sqrt{\tfrac{2}{5}}c, 0\right). \tag{S.44}$$

(b) The proper velocity 4–vector is related to the 3–velocity as

$$u^\mu \;=\; \gamma(c, v^x, v^y, v^z) \tag{S.45}$$

where for the particle at hand

$$\gamma \;=\; \frac{1}{\sqrt{1-(v/c)^2}} \;=\; \frac{1}{\sqrt{1-\frac{4}{5}}} \;=\; \sqrt{5}. \tag{S.46}$$

Consequently,

$$u^\mu \;=\; \left(u^0, u^x, u^y, u^z\right) \;=\; \left(\sqrt{5}c, \sqrt{2}c, \sqrt{2}c, 0\right). \tag{S.47}$$

(c) For the new frame K' moving at speed $v_{\text{rel}} = \sqrt{\frac{2}{5}}c$ in the positive x direction, we have

$$\beta_{\text{rel}} \;=\; \sqrt{\tfrac{2}{5}}\,, \qquad \gamma_{\text{rel}} \;=\; \sqrt{\tfrac{5}{3}}, \qquad \gamma_{\text{rel}}\beta_{\text{rel}} \;=\; \sqrt{\tfrac{2}{3}}\,, \tag{S.48}$$

hence the Lorentz transform matrix from the K frame to the K' frame is

$$L^\mu_{\ \nu} \;=\; \begin{pmatrix} \sqrt{\frac{5}{3}} & -\sqrt{\frac{2}{3}} & 0 & 0 \\ -\sqrt{\frac{2}{3}} & \sqrt{\frac{5}{3}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{S.49}$$

In particular, the proper velocity 4–vector (S.47) transforms to

$$u'^\mu \;=\; \begin{pmatrix} \sqrt{\frac{5}{3}} & -\sqrt{\frac{2}{3}} & 0 & 0 \\ -\sqrt{\frac{2}{3}} & \sqrt{\frac{5}{3}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{5} \\ \sqrt{2} \\ \sqrt{2} \\ 0 \end{pmatrix} c \;=\; \begin{pmatrix} \sqrt{3} \\ 0 \\ \sqrt{2} \\ 0 \end{pmatrix} c. \tag{S.50}$$

In other words,

$$\left(u'^0, u'^x, u'^y, u'^z\right) \;=\; \left(\sqrt{3}c, 0, \sqrt{2}c, 0\right). \tag{S.51}$$

(d) By inspection,

$$u'^{\mu}u'_{\mu} \;=\; (u'^{0})^2 \;-\; (\mathbf{u}')^2 \;=\; 3c^2 \;-\; 2c^2 \;=\; c^2. \tag{S.52}$$

(e)

$$\mathbf{v}' \;=\; \frac{c}{u'^{0}}\,\mathbf{u}' \;=\; \frac{1}{\sqrt{3}}\big(0,\sqrt{2}c,0\big) \;=\; \big(0,\sqrt{\tfrac{2}{3}}c,0\big). \tag{S.53}$$

Problem **7**:

Consider the total energy and the total 3–momentum of the n-particle system,

$$\begin{aligned} E_{\mathrm{net}} \;&=\; E_1 \;+\; \cdots \;+\; E_n\,, \\ \mathbf{P}_{\mathrm{net}} \;&=\; \mathbf{p}_1 \;+\; \cdots \;+\; \mathbf{p}_n\,. \end{aligned}$$

Together $P^0_{\mathrm{net}} = \frac{1}{c}E_{\mathrm{net}}$ and $\mathbf{P}_{\mathrm{net}}$ form a 4–vector P^{μ}_{net}, which under Lorentz transforms between different inertial frames behaves as any other 4–vector,

$$P'^{\mu}_{\mathrm{net}} \;=\; L^{\mu}_{\ \nu}P^{\nu}_{\mathrm{net}}\,. \tag{S.54}$$

Also, this vector is timelike, $(P_{\mathrm{net}}\cdot P_{\mathrm{net}}) > 0$. Indeed, for each individual particle $(p_a\cdot p_a) = m_a^2c^2 > 0$, hence

$$\text{each } \frac{1}{c}E_a \;>\; |\mathbf{p}_a|. \tag{S.55}$$

Consequently,

$$|\mathbf{P}_{\mathrm{net}}| \;=\; \left|\sum_a \mathbf{p}_a\right| \;\le\; \sum_a |\mathbf{p}_a| \;<\; \sum_a \frac{1}{c}E_a \;=\; \frac{1}{c}E_{\mathrm{net}} \;=\; P^0_{\mathrm{net}}\,, \tag{S.56}$$

and therefore

$$\big(P_{\mathrm{net}}\cdot P_{\mathrm{net}}\big) \;=\; \big(P^0_{\mathrm{net}}\big)^2 \;-\; \big(\mathbf{P}_{\mathrm{net}}\big)^2 \;>\; 0. \tag{S.57}$$

The bottom line is, P^{μ}_{net} is a time-like 4–vectors, and for any such vector there is an inertial frame where its space component $\mathbf{P}_{\mathrm{net}}$ happens to vanish. Specifically, this frame moves relative to K frame at velocity

$$\mathbf{v} \;=\; c\frac{\mathbf{P}_{\mathrm{net}}}{P^0_{\mathrm{net}}} \;=\; c^2\frac{\mathbf{P}_{\mathrm{net}}}{E_{\mathrm{net}}}\,. \tag{S.58}$$

Note that this velocity is slower than c because $|\mathbf{P}_{\mathrm{net}}| < P^0_{\mathrm{net}}$, so the inertial frame K' moving at this velocity does exist. And in that frame K',

$$\begin{aligned}\mathbf{P}'_{\mathrm{net}} \;&=\; -\gamma\boldsymbol{\beta}P^0_{\mathrm{net}} \;+\; \mathbf{P}_{\mathrm{net}} \;+\; \frac{\gamma-1}{\beta^2}(\boldsymbol{\beta}\cdot\mathbf{P}_{\mathrm{net}})\boldsymbol{\beta} \\ &\qquad \text{for } \boldsymbol{\beta} \;=\; \frac{\mathbf{P}_{\mathrm{net}}}{P^0_{\mathrm{net}}} \\ &=\; -\gamma\mathbf{P}_{\mathrm{net}} \;+\; \mathbf{P}_{\mathrm{net}} \;+\; (\gamma-1)\mathbf{P}_{\mathrm{net}} \\ &=\; 0.\end{aligned} \tag{S.59}$$

Problem **8**:

The first particle has $E_1 = 2\times mc^2$, which means $\gamma = 2$, hence

$$\beta\gamma \;=\; \sqrt{\gamma^2-1} \;=\; \sqrt{3} \tag{S.60}$$

and therefore 3–momentum $p_1 = \sqrt{3}mc$. The second particle is at rest, thus $E_2 = mc^2$ (for the same m since it's of the same species as the first particle) and $p_2 = 0$. Altogether, the two particles have net energy

$$E_{\mathrm{net}} \;=\; E_1 \;+\; E_2 \;=\; 3mc^2 \tag{S.61}$$

and net 3–momentum

$$P_{\mathrm{net}} \;=\; p_1 \;+\; p_2 \;=\; \sqrt{3}mc. \tag{S.62}$$

Both the net energy and the net momentum are conserved in the collision, so if the two particles stick together in the collision and form a combined particle, then it has

$$E = 3mc^2 \quad \text{and} \quad p = \sqrt{3}mc, \tag{S.63}$$

or in terms of the energy-momentum 4–vector

$$P^\mu = (3, \sqrt{3}, 0, 0)mc. \tag{S.64}$$

The rest mass M of this combined particles obtains from

$$M^2c^2 = (P \cdot P) = (9-3)m^2c^2, \tag{S.65}$$

hence $M = \sqrt{6}\,m \approx 2.45\,m$. Finally, the 3–velocity obtains from the energy–momentum 4-vector as

$$\mathbf{v} = c\frac{\mathbf{P}}{P^0} = \frac{c\sqrt{3}}{3} = \frac{c}{\sqrt{3}}\,. \tag{S.66}$$

Problem **9**:

In the frame K_π of the original pion, both photons have equal energies and equal and opposite momenta,

$$\begin{aligned} E_1 &= E_2 = \tfrac{1}{2}m_\pi c^2 = 67.7\text{ MeV}, \\ \mathbf{p}_1 &= -\mathbf{p}_2, \\ |\mathbf{p}_1| &= |\mathbf{p}_2| = \frac{E_1 = E_2}{c} = \tfrac{1}{2}m_\pi c = 67.5\text{ MeV}/c. \end{aligned} \tag{S.67}$$

In the lab frame, the two photons' energies obtain from the Lorentz boost from the pion's frame,

$$\begin{aligned} E_1^{\text{lab}} &= \gamma_\pi(E_1 + \mathbf{v}_\pi \cdot \mathbf{p}_1), \\ E_2^{\text{lab}} &= \gamma_\pi(E_2 + \mathbf{v}_\pi \cdot \mathbf{p}_2) = \gamma_\pi(E_1 - \mathbf{v}_\pi \cdot \mathbf{p}_1). \end{aligned} \tag{S.68}$$

In particular, since the first photon continues in the forward direction while the second photon flies back,

$$\begin{aligned} E_1^{\text{lab}} &= \gamma_\pi\left(E_1 + v_\pi\frac{E_1}{c}\right) = \gamma_\pi(1+\beta_\pi)\times\frac{m_\pi c^2}{2}\,, \\ E_2^{\text{lab}} &= \gamma_\pi(1-\beta_\pi)\times\frac{m_\pi c^2}{2}\,. \end{aligned} \tag{S.69}$$

Finally, for the pion in question $p = \frac{3}{4}mc$, which means $\beta_\pi\gamma_\pi = \frac{3}{4}$ and therefore

$$\gamma_\pi = \sqrt{1+(\beta\gamma)^2} = \sqrt{1+\tfrac{9}{16}} = \tfrac{5}{4}. \tag{S.70}$$

Consequently,

$$\gamma_\pi(1+\beta_\pi) = \tfrac{5}{4} + \tfrac{3}{4} = 2, \qquad \gamma_\pi(1-\beta_\pi) = \tfrac{5}{4} - \tfrac{3}{4} = \tfrac{1}{2}, \tag{S.71}$$

and therefore

$$E_1^{\text{lab}} = m_\pi c^2 = 135\ \text{MeV}, \qquad E_2^{\text{lab}} = \tfrac{1}{4}m_\pi c^2 \approx 34\ \text{MeV}. \tag{S.72}$$

<u>Problem</u> **10** is postponed to the next homework set.