Physics 303K course summary (11/24/99)

This sheet contains most of the *basic* formulas in Physics 303K. This sheet is intended to be a guide for doing homework problems, for reviewing for the exams, and to be a reference for Physics 303L and for future use. **During any exam, questions on this sheet will not be answered.** However, at other times your questions are welcome.

Mathematics (§3, §7, §11, Appendix B)

- For $ax^2+bx+c=0, x=(-b\pm\sqrt{b^2-4ac})/(2a)$.
- Cartesian and polar coordinates:
  $x=r\cos\theta,\ y=r\sin\theta$
  $r^2=x^2+y^2,\ \tan\theta=\frac{y}{x}$
- Trig : $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
  $\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
  $\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
  $\sin 2\theta=2\sin\theta\cos\theta, \cos 2\theta=\cos^2\theta-\sin^2\theta$
  $1-\cos\theta=2\sin^2\frac{\theta}{2},\ 1+\cos\theta=2\cos^2\frac{\theta}{2}$.
- Vector algebra: $\vec{A}=(A_x,A_y)=A_x\hat{\imath}+A_y\hat{\jmath}$
  Resultant: $\vec{R}=\vec{A}+\vec{B}=(A_x+B_x,A_y+B_y)$
  Method of parallelgram: R is diagonal.
- $\vec{A}\cdot\vec{B}=AB\cos\theta=A_xB_x+A_yB_y+A_zB_z$
- Cross product: $\hat{\imath}\times\hat{\jmath}=\hat{k},\hat{\jmath}\times\hat{k}=\hat{\imath},\hat{k}\times\hat{\imath}=\hat{\jmath}$.

$$\vec{C}=\vec{A}\times\vec{B}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

$C=AB\sin\theta,\ =A_\perp B=AB_\perp,$ right hand rule.

Calculus: $\frac{dx^n}{dx}=nx^{n-1}, \frac{dlnx}{dx}=\frac{1}{x}$
$\frac{d\sin\theta}{d\theta}=\cos\theta,\ \frac{d\cos\theta}{d\theta}=-\sin\theta$

Measurements (§1)

- SI units; Standards of length, mass, time.
- Dimensional analysis: In any physics expression, dimension of each item must be the same.
  $e.g.[F]=[m][a]=MLT^{-2}, F=m^iv^jr^k$.
- Signif. figures: $c=ab, sf(c)=Min[sf(a),sf(b)]$
- Summation: $\sum_{i=1}^N(ax_i+b)=a\sum_{i=1}^N x_i+bN$

Motion(§2,4)

- One dim motion: $v=\frac{ds}{dt}, a=\frac{dv}{dt}$
  Average values: $\bar{v}=\frac{s_f-s_i}{t_f-t_i}, \bar{a}=\frac{v_f-v_i}{t_f-t_i}$
- One dim motion with constant acceleration:
  $vt:\quad v=v_0+at$
  $st:\quad s=\bar{v}t=v_0t+\frac{1}{2}at^2$, $\bar{v}=\frac{v_0+v}{2}$.
  $vs:\quad v^2=v_0^2+2as$
- Nonunif. acc.: Method of separation of variables.
- The independent x and y motion
- Projectile motion: $t_{rise}=t_{fall}=\frac{t_{trip}}{2}=\frac{v_{0y}}{g}$
  $h=\frac{1}{2}gt_{fall}^2,\ R=v_{ox}t_{trip}$
  Circular: $a_c=\frac{v^2}{r}, v=\frac{2\pi r}{T}, f=\frac{1}{T}$ (Hertz)

Curvilinear motion: $a=\sqrt{a_t^2+a_r^2}$.

- Relative velocity: $\vec{v}=\vec{v}'+\vec{u}$

Law of Motion and applications (§5,6)

- Force: $\vec{F}=m\vec{a},\ W=mg,\ \vec{F}_{12}=-\vec{F}_{21}$.
- Circular motion: $a_c=\frac{v^2}{r},\ v=\frac{2\pi r}{T}=2\pi rf$.
- Friction: $f_s\le\mu_sN, f_k=\mu_kN$
- Gravity: $F=\frac{Gm_1m_2}{d^2}$. On earth : $g=9.8$ m/s^2,
- Equilibrium (concurrent forces): $\Sigma_i\vec{F}_i=0$.

Energy (§7-8)

- Work (for all F) :$W_{A\to B}=Fs_\parallel=F_\parallel s=Fs\cos\theta$
  $=\vec{F}\cdot\vec{s}\to\int_A^B\vec{F}\cdot d\vec{s}$ (in Joules)
- Three types of effects due to work done:
  $F^{ext}=ma+F+f$
  $W^{ext}|_{A\to B}=(K_B-K_A)+(U_B-U_A)+W_{diss}|_{A\to B}$
- Kinetic energy: $\mathrm{K}_B-\mathrm{K}_A=\int_A^B F(x)dx,\ K=\frac{1}{2}mv^2$
- PE (conservative $\vec{F}$): $U_B-U_A=-\int_A^B\vec{F}\cdot d\vec{s}$
- $U_{gravity}=mgy,\ U_{spring}=\frac{1}{2}kx^2$
- From U to $\vec{\mathrm{F}}$: $F_x=-\frac{\partial U}{\partial x},\ F_y=-\frac{\partial U}{\partial y}\ldots$
- $F_{gravity}=-\frac{\partial U}{\partial y}=-mg, F_{spring}=-\frac{\partial U}{\partial x}=-kx$.
- Equilibrium : $\frac{\partial U}{\partial x}=0, \frac{\partial^2U}{\partial x^2}>0$ (stable), <0 (unst.)
- Power: $P=\frac{dW}{dt}=Fv_\parallel=Fv\cos\theta=\vec{F}\cdot\vec{v}$. $(watts)$

Collision (§9)

- Impulse :$\vec{I}=\Delta\vec{p}=\vec{p}_f-\vec{p}_i\to\int_{t_i}^{t_f}\vec{F}dt$
- Momentum : $\vec{p}=m\vec{v}$
- Two-body,1 dim : $x_{cm}=\frac{m_1x_1+m_2x_2}{M}, M=m_1+m_2$
  $P_{cm}\equiv Mv_{cm}=p_1+p_2$
  $F_{cm}\equiv F_1+F_2=m_1a_1+m_2a_2=Ma_{cm}$
  $K_1+K_2=K_1^*+K_2^*+K_{cm}$
- Two-body collision: $\vec{P}_i=\vec{P}_f=(m_1+m_2)\vec{v}_{cm}$
  $v_i^*=v_i-v_{cm},\ v_i'=v_i^{*\prime}+v_{cm}$
- Elastic: $v_1-v_2=-(v_1'-v_2')$,
  $v_i^{*\prime}=-v_i^*, v_i'=2v_{cm}-v_i$
- Many body, center of mass:
  $\vec{r}_{cm}=\frac{\Sigma_im_i\vec{r}_i}{M}=\frac{\int\vec{r}dm}{M},\ M=\Sigma_im_i$
- Force on cm : $\vec{F}^{ext}=\frac{d\vec{P}}{dt}=M\vec{a}_{cm},\ \vec{P}=\Sigma_i\vec{p}_i$

Rotation of Rigid-Body (§10)

- Kinematics: $\theta=\frac{s}{r},\ \omega=\frac{v}{r},\ \alpha=\frac{a_t}{r}$.
- Moment of inertia: $I=\Sigma_im_ir_i^2=\int r^2dm$
  $I_{disk}=\frac{1}{2}MR^2, I_{ring}=\frac{1}{2}M(R_1^2+R_2^2)$
  $I_{rod}=\frac{1}{12}M\ell^2,\ I_{rectangle}=\frac{1}{12}M(a^2+b^2)$
  $I_{sphere}=\frac{2}{5}MR^2,\ I_{sph-shell}=\frac{2}{3}MR^2$
  $I=M*(\text{Radius of gyration})^2$
  $I=I_{cm}+MD^2$

- Kinetic energies: $K_{rot} = \frac{1}{2}I\omega^2, K = K_{rot} + K_{cm}$
- Ang.momentum: $L = rmv \to \sum_i r_i m_i(\omega r_i) = I\omega$
- Torque: $\tau = \frac{dL}{dt} = m\frac{dv}{dt}r = Fr \to I\frac{d\omega}{dt} = I\alpha$
- $W^{ext} = \Delta K + \Delta U + W_f, K = K_{rot} + \frac{1}{2}mv^2,\ P = \tau\omega$

Rolling, angular momentum and torque (§11)

- Rolling: $K = \frac{1}{2}(I_c + MR^2)\omega^2 = \frac{1}{2}(\frac{I_c}{R^2} + M)v^2$
- Angular momentum: $\vec{L} = \vec{r} \times \vec{p},\ L = r_\perp p \to I\omega.$
- Torque: $\vec{\tau} = \frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \vec{F}, \tau = r_\perp F \to I\alpha.$
- Gyroscope: $\omega_p = \frac{d\phi}{dt} = \frac{1}{L}\frac{dL}{dt} = \frac{\tau}{L} = \frac{mgh}{I\omega}$

Static equilibrium (§12)

- $\Sigma\vec{F}_i = 0,$ about any point $\Sigma\vec{\tau}_i = 0.$
- Subdivisions: $\vec{r}_{cm} = \frac{m_A\vec{r}_{Acm} + m_B\vec{r}_{Bcm}}{m_A + m_B}$
- Elastic modulus=stress/strain

  stress: F/A

  strain: $\Delta L/L,\ \theta \approx \Delta x/h,\ -\Delta V/V.$

Oscillation motion (§13) $f = \frac{1}{T},\ \omega = \frac{2\pi}{T}, \lambda = vT.$

- SHM : $a = \frac{d^2x}{dt^2} = -\omega^2 x,\ \alpha = \frac{d^2\theta}{dt^2} = -\omega^2\theta$

  $x = x_{max}\cos(\omega t + \delta)$, $x_{max} = A$

  $v = -v_{max}\sin(\omega t + \delta)$, $v_{max} = \omega A$

  $a = -a_{max}\cos(\omega t + \delta) = -\omega^2 x,\ a_{max} = \omega^2 A$
- $E = K + U = K_{max} = \frac{1}{2}m(\omega A)^2 = U_{max} = \frac{1}{2}kA^2.$
- Spring : $ma = -kx$
- Simple pendulum : $a_t = \alpha\ell = -g\sin\theta$
- Physical pendulum: $\tau = I\alpha = -mgl\sin\theta$
- Torsion pendulum : $\tau = I\alpha = -\kappa\theta.$

Gravity (§14)

- At $r \geq R$: $g(r) = \frac{GM}{r^2} = g\left(\frac{R}{r}\right)^2$
- Circular orbit: $a_c = \frac{v^2}{r} = \omega^2 r = \left(\frac{2\pi}{T}\right)^2 r = g(r)$

  $U = -\frac{GmM}{r}$, $F = -\frac{dU}{dr} = -\frac{GmM}{r^2},\ \frac{mv^2}{r} = \frac{GmM}{r^2}$

  $G = 6.67 \times 10^{-11} Nm^2/kg^2, R_{earth} = 6370$ km .

  $E = U + K = -\frac{GmM}{2r}.$
- Kepler's Laws of planetary motion

  i. Elliptical orbit: Sun at one focal point.

  $r = \frac{r_0}{1 - \epsilon\cos\theta},\ r_1 = \frac{r_0}{1+\epsilon}\ r_2 = \frac{r_0}{1-\epsilon}.$

  ii. $L = rm\frac{\Delta r_\perp}{\Delta t} \to \frac{\Delta A}{\Delta t} = \frac{1}{2}\frac{r\Delta r_\perp}{\Delta t} = \frac{L}{2m} =$ const.

  iii. $\frac{GM}{r^2} = \left(\frac{2\pi r}{T}\right)^2\frac{1}{r} \to \frac{GM}{a^2} = \left(\frac{2\pi a}{T}\right)^2\frac{1}{a}.$

($a = \frac{r_1 + r_2}{2}$ = semimajor axis of the elliptic orbit.)

- Escape kinetic energy: $E = K + U(R) = 0.$

Fluid mechanics (§15)

- Pascal: $P = \frac{F_{\perp 1}}{A_1} = \frac{F_{\perp 2}}{A_2},\ 1atm = 1.013 \times 10^5 N/m^2.$
- Archimedes: $B = Mg$. Pascal $= N/m^2$.
- $P = P_{atm} + \rho gh$, with $P = \frac{F_\perp}{A}$ and $\rho = \frac{m}{V}$.
- $F = \int PdA \to \rho g\ell\int_0^h (h - y)dy.$
- Continuity equation: Av=constant.
- Bernoulli: $P + \frac{1}{2}\rho v^2 + \rho$gy=const. $P \geq 0.$

Wave motion (§16)

- Traveling waves: $y = f(x - vt), y = f(x + vt).$

  Right moving: $y = A\sin(kx - \omega t - \phi)$
- Along a string: $v = \sqrt{\frac{T}{\mu}}$
- General: $\Delta E = \Delta K + \Delta U = \Delta K_{max}$

- 1 dim waves: $\mathcal{P} = \frac{\Delta E}{\Delta t} = \frac{1}{2}A^2\omega^2\mu v$

†• Circular: $\frac{\Delta m}{\Delta t} = \frac{\Delta m}{\Delta A} \cdot \frac{\Delta A}{\Delta r} \cdot \frac{\Delta r}{dt} = \frac{\Delta m}{\Delta A} \cdot 2\pi rv$

†• Spherical: $\frac{\Delta m}{\Delta t} = \frac{\Delta m}{\Delta V} \cdot 4\pi r^2 v$

Sound (§17)

$v = \sqrt{\frac{B}{\rho}}$, $s = s_{max}\cos(kx - \omega t - \phi)$

$\Delta P = -B\frac{\Delta V}{V} = -B\frac{\partial s}{\partial x}$

$\Delta P_{max} = B\kappa s_{max} = \rho v\omega s_{max}$

- Piston: $\frac{\Delta m}{\Delta t} = \frac{\Delta m}{\Delta V} \cdot \frac{A\Delta x}{\Delta t} = \rho Av$
- Intensity: $I = \frac{P}{A} = \frac{1}{2}\rho v(\omega s_{\max})^2$
- Intensity level: $\beta = 10\log_{10}\frac{I}{I_0}$, $I_0 = 10^{-12}\frac{W}{m^2}$
- Plane waves: $\psi(x, t) = c\sin(kx - \omega t)$

†• Circular waves: $\psi(r, t) = \frac{c}{\sqrt{r}}\sin(kr - \omega t)$

†• Spherical: $\psi(r, t) = \frac{c}{r}\sin(kr - \omega t)$

- Doppler effect: $\lambda = vT,\ f_0 = \frac{1}{T},\ f' = \frac{v'}{\lambda'}.$

  Here $v' = v_{sound} \pm v_{observer}$, is wave speed relative to moving observer and

  $\lambda' = (v_{sound} \pm v_{source})/f_0$, detected wave length established by moving source of frequency f_0.
- Received and reflected frequencies are the same.
- Shock waves: Mach Number= $\frac{v_{source}}{v_{sound}} = \frac{1}{\sin\theta}$

Superposition of waves (§18)

- Phase difference: $\sin(kx - \omega t) + \sin(kx - \omega t - \phi)$
- Standing waves: $\sin(kx - \omega t) + \sin(kx + \omega t)$
- Beats: $\sin(kx - \omega_1 t) + \sin(kx - \omega_2 t)$

†• Others: $a\cos(kx - \omega t) + b\sin(kx - \omega t).$

$y = \sin(kx - \omega t),\ z = \sin(kx - \omega t).$

- Fundamental modes: Sketch wave patterns.

  Distance between adjacent nodes: $\lambda/2$.

  String: $\frac{\lambda}{2} = L;\ f = \frac{\sqrt{F/\mu}}{2L}.$

  Open-open pipe: $\frac{\lambda}{2} = L;\ f = \frac{v_s}{2L}.$

  Open-closed pipe: $\frac{\lambda}{4} = L;\ f = \frac{v_s}{4L}.$

  Rod clamped at middle: $\frac{\lambda}{2} = L$. Higher order modes: Sketch wave patterns.

  Two nodes with n antinodes between them: $L = n\lambda/2$.

  String: $\frac{n\lambda}{2} = L;\ f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}},\ n = 1, 2, 3, \ldots.$

  Open-open pipe: $\frac{n\lambda}{2} = L;\ f_n = \frac{nv_s}{2L},\ n = 1, 2, 3, \ldots.$

  Open-closed pipe: $\frac{n\lambda}{4} = L;\ f_n = \frac{nv_s}{4L},\ n = 1, 3, 5, \ldots.$

  Rod clamped at middle: $\frac{n\lambda}{2} = L,\ n = 1, 3, 5, \ldots.$